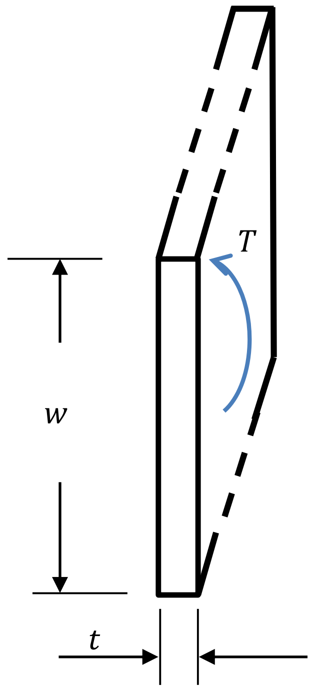
56. A torque $T \sim N(10, 1^2)$ kN·m is applied to a steel strip, as shown in the figure. The strip has a length of $l = 300$ mm and a thickness of $t = 3$ mm. If the allowable shear stress of the steel is $\tau_a \sim (100, 10^2)$ MPa and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum width of the steel using membrane analogy theory.



Solution

According to the membrane analogy theory,

$$\tau = G\theta_1 c = \frac{3T}{Lc^2} = \frac{3T}{wt^2}$$

where τ is the shear stress, G is the shear modulus, θ_1 is the angle of twist per unit length, T is the torque, L is the length of the median line, and c is the wall thickness.

The limit-state function is the shear stress of the strip subtracted from the allowable shear stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{3T}{wt^2}$$

where $\mathbf{X}$=(τ_a, T).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{\tau_a} - \frac{3}{wt^2}\mu_T$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial \tau_a}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{\tau_a}\right)^2 + \left(\left.\frac{\partial g}{\partial T}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_T\right)^2} = \sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{3}{wt^2}\sigma_T\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\mu_{\tau_a} - \frac{3}{wt^2}\mu_T\right)}{\sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{3}{wt^2}\sigma_T\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\mu_{\tau_a} - \frac{3}{wt^2}\mu_T\right)}{\sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{3}{wt^2}\sigma_T\right)^2}}$$

$$= \frac{-\left(100(10^6) - \frac{3}{w(3(10^{-3}))^2}(10)(10^3)\right)}{\sqrt{\left(10(10^6)\right)^2 + \left(-\frac{3}{w(3(10^{-3}))^2}(1)(10^3)\right)^2}}$$

Solving for w yields

$$w = 64.17 \quad \text{mm}$$

Ans.