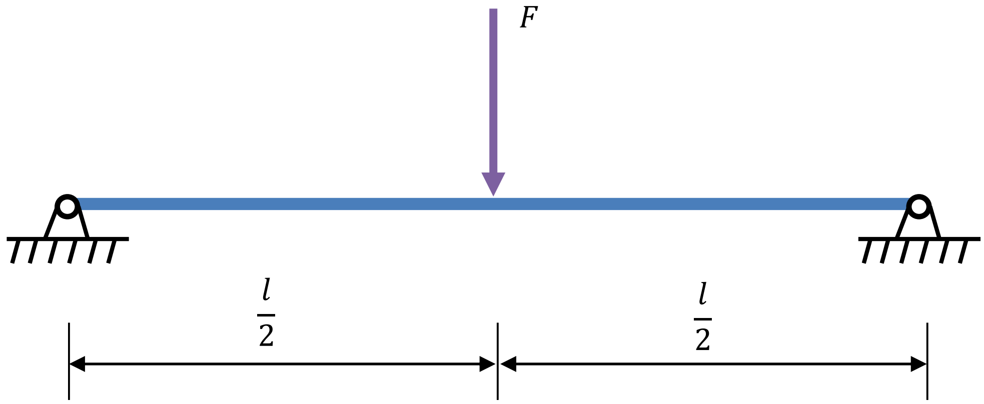
57. A simply-supported beam is subjected to a force $F \sim N(300,\ 30^2)$ lbf, as shown in the figure. The length of the beam is $l \sim N(30,\ 0.1^2)$ in and the modulus of elasticity is $E = 12$ Mpsi. If the allowable deflection is $\delta_a = 0.5$ in and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of the beam using $y_{\max} = \frac{Fl^3}{48EI}$. Assume that F and l are independent.



Solution

The maximum deflection of the beam is

$$y_{\max} = \frac{Fl^3}{48EI}$$

Thus the limit-state function is the maximum deflection subtracted from the allowable deflection. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - y_{\max} = \delta_a - \frac{Fl^3}{48EI}$$

where $\mathbf{X}=(F, l)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \delta_a - \frac{\mu_F \mu_l^3}{48EI}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_F\right)^2 + \left(\left.\frac{\partial g}{\partial l}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_l\right)^2} = \sqrt{\left(-\frac{\mu_l^3}{48EI}\sigma_F\right)^2 + \left(-\frac{3\mu_F \mu_l^2}{48EI}\sigma_l\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\delta_a - \frac{\mu_F \mu_l^3}{48EI}\right)}{\sqrt{\left(-\frac{\mu_l^3}{48EI}\sigma_F\right)^2 + \left(-\frac{3\mu_F \mu_l^2}{48EI}\sigma_l\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\delta_a - \frac{\mu_F \mu_l^3}{48EI}\right)}{\sqrt{\left(-\frac{\mu_l^3}{48EI}\sigma_F\right)^2 + \left(-\frac{3\mu_F \mu_l^2}{48EI}\sigma_l\right)^2}}$$

$$= \frac{-\left(0.5 - \frac{300}{48(12)(10^6)\frac{\pi}{64}d^4}30^3\right)}{\sqrt{\left(-\frac{30^3}{48(12)(10^6)\frac{\pi}{64}d^4}(30)\right)^2 + \left(-\frac{3(300)30^2}{48(12)(10^6)\frac{\pi}{64}d^4}(0.1)\right)^2}}$$

Solving for d yields

$$d = 0.951 \quad \text{in}$$

Thus $d = 1.0$ in can be used.

Ans.