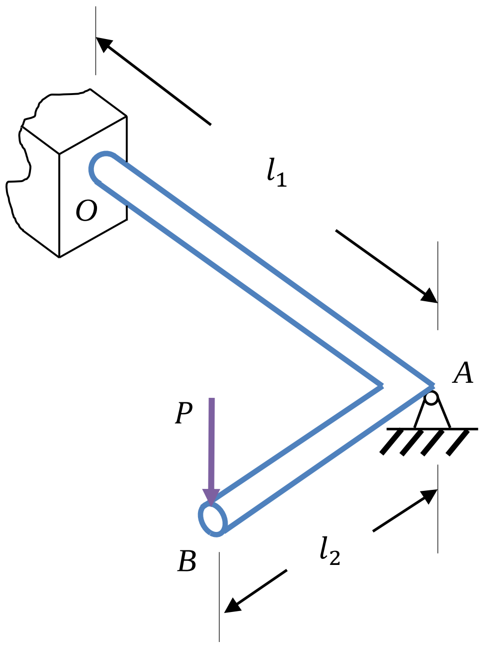
58. A torsion bar *OA* is fixed at *O* and is simply supported at *A*. It is connected to a cantilever *AB*, and a force $P \sim N(1000, 50^2)$ N is applied at *B*. Bar *OA* has a spring rate of $k_1 = 2 \times 10^5$ N·m/rad and a length of $l_1 \sim N(0.5, 0.001^2)$ m. Cantilever *AB* has a spring rate of $k_2 = 3 \times 10^4$ N/m and a length of $l_2 \sim N(0.2, 0.001^2)$ m. If the allowable deflection at *B* is $\delta_a = 0.04$ m, determine the probability of failure using First Order Second Moment Method. Assume that *P*, l_1 and l_2 are independent.



Solution

The torque of torsion bar *OA* is

$$M_A = Pl_2$$

Thus the angle of twisting is

$$\theta_A = \frac{M_A}{k_1} = \frac{Pl_2}{k_1}$$

Then the deflection δ_1 at *B* resulted from the twisting of torsion bar *OA* is

$$\delta_1 = \theta_A l_2 = \frac{Pl_2^2}{k_1}$$

For the cantilever *BC*, the deflection δ_2 at *B* resulted from force *P* is

$$\delta_2 = \frac{P}{k_2}$$

Then the overall deflecting at *B* is

$$\delta = \delta_1 + \delta_2 = \frac{Pl_2^2}{k_1} + \frac{P}{k_2}$$

The limit-state function is the actual deflection subtracted from the allowable deflection. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - \delta = \delta_a - \frac{Pl_2^2}{k_1} - \frac{P}{k_2}$$

where $\mathbf{X}$=(P, l_2).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \delta_a - \frac{\mu_P \mu_{l_2}^2}{k_1} - \frac{\mu_P}{k_2} = 0.04 - \frac{1000(0.2)^2}{2(10^5)} - \frac{1000}{3(10^4)} = 6.467(10^{-3}) \quad \text{m}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2 + \left(\left.\frac{\partial g}{\partial l_2}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{l_2}\right)^2}$$

$$= \sqrt{\left(\left(-\frac{\mu_{l_2}^2}{k_1} - \frac{1}{k_2}\right)\sigma_P\right)^2 + \left(2\frac{\mu_P \mu_{l_2}}{k_1}\sigma_{l_2}\right)^2}$$

$$= \sqrt{\left(\left(-\frac{(0.2)^2}{2(10^5)} - \frac{1}{3(10^4)}\right)(50)\right)^2 + \left(2\frac{1000(0.2)}{2(10^5)}(0.001)\right)^2}$$

$$= 1.677(10^{-3}) \qquad \text{m}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{6.467(10^{-3})}{1.677(10^{-3})}\right) = 5.74(10^{-5})$$

Ans.