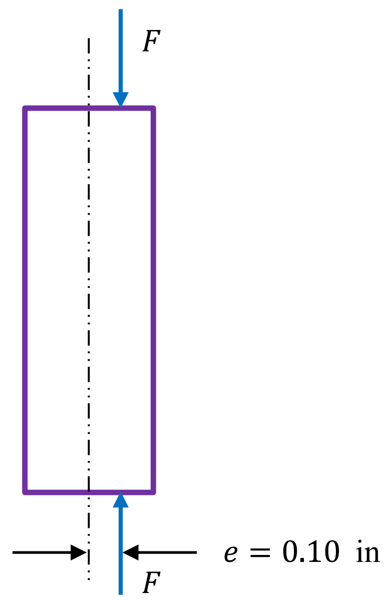
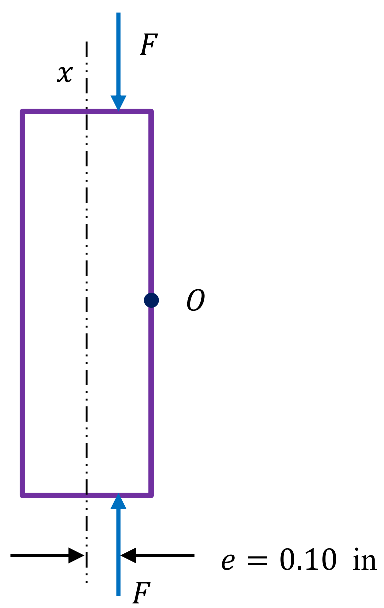
59. An eccentric force $F \sim N(900,\ 90^2)$ lbf is applied to a strut with a square cross section shown in the figure. The yield strength of the strut is $S_y \sim N(2000,\ 200^2)$ psi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum side length of the square cross section using the theory of short compression member. Assume that $F$ and $S_y$ are independent.




**Solution**




Based on the theory of short compression member, the maximum compressive stress occurs at point $O$, and is given by

$$S_{max} = \frac{F}{A}\left(1 + \frac{ecA}{I}\right)$$

where $A$ is the cross-section area of the strut, $c$ is the distance between point $O$ and axis $x$, and $I$ is the moment of inertia.

The limit-state function is the maximum compression stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S_{max} = S_y - \frac{F}{A}\left(1 + \frac{ecA}{I}\right)$$

where $\mathbf{X}=(S_y, F)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_y} - \frac{1}{A}\left(1 + \frac{ecA}{I}\right)\mu_F$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_F\right)^2} = \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{1}{A}\left(1 + \frac{ecA}{I}\right)\sigma_F\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\mu_{S_y} - \frac{1}{A}\left(1 + \frac{ecA}{I}\right)\mu_F\right)}{\sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{1}{A}\left(1 + \frac{ecA}{I}\right)\sigma_F\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\mu_{S_y} - \frac{1}{A}\left(1 + \frac{ecA}{I}\right)\mu_F\right)}{\sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{1}{A}\left(1 + \frac{ecA}{I}\right)\sigma_F\right)^2}}$$

$$= \frac{-\left(2000 - \frac{1}{b^2}\left(1 + \frac{(0.1)\left(\frac{b}{2}\right)b^2}{\frac{b(b^3)}{12}}\right)(900)\right)}{\sqrt{(200)^2 + \left(-\frac{1}{b^2}\left(1 + \frac{(0.1)\left(\frac{b}{2}\right)b^2}{\frac{b(b^3)}{12}}\right)(90)\right)^2}}$$

Solving for $b$ yields

$$b = 1.15 \text{ in}$$

Thus $b = 1.20$ in can be used.

**Ans.**