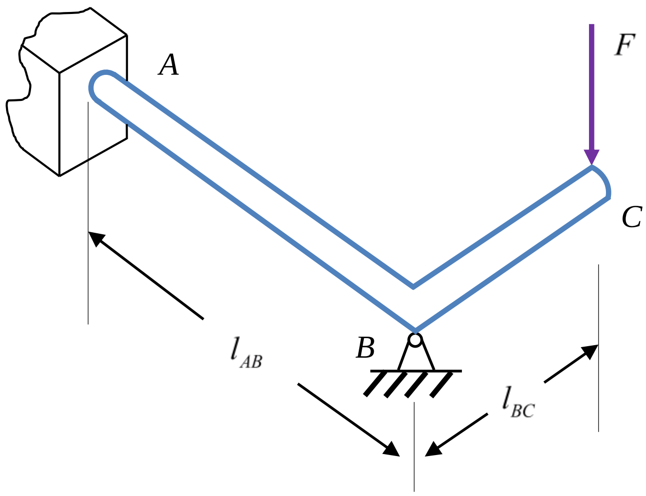
6. A torsion bar *AB* is supported at *B* and is fixed at *A*. The bar is connected to a cantilever *BC*. The spring rate of bar *AB* is $k_{AB} \sim N(\mu_{AB}, \sigma_{AB}{}^2)$, and the spring rate of the cantilever *BC* is $k_{BC} \sim N(\mu_{BC}, \sigma_{BC}{}^2)$. If k_{AB} and k_{BC} are independent, what is the overall spring rate with respect to the vertical deflection δ at *C*? Find its mean and standard deviation using the First Order Second Moment Method.



Solution

For the torsion bar *AB*, the acting torque is

$$M_B = Fl_{BC} \tag{1}$$

Thus the angle of twisting is

$$\theta_B = \frac{M_B}{k_{AB}} = \frac{Fl_{BC}}{k_{AB}} \tag{2}$$

Then the deflection δ_1 at *C* resulted from the twisting of torsion bar *AB* is

$$\delta_1 = \theta_B l_{BC} = \frac{Fl_{BC}^2}{k_{AB}} \tag{3}$$

For the cantilever *BC*, the deflection δ_2 at *C* resulted from force *F* is

$$\delta_2 = \frac{F}{k_{BC}} \tag{4}$$

Then the overall deflecting at C is

$$\delta = \delta_1 + \delta_2 = \frac{Fl_{BC}^2}{k_{AB}} + \frac{F}{k_{BC}} \tag{5}$$

Solving for the overall spring rate k

$$k = g(k_{AB},\, k_{BC}) = \frac{F}{\frac{Fl_{BC}^2}{k_{AB}} + \frac{F}{k_{BC}}} = \frac{1}{\frac{l_{BC}^2}{k_{AB}} + \frac{1}{k_{BC}}} = \frac{k_{AB}k_{BC}}{k_{AB} + k_{BC}l_{BC}^2} \tag{6}$$

Ans.

Now using FOSM, we have

$$\mu_k = g(\mu_{k_{AB}},\, \mu_{k_{BC}}) = g(\mu_{AB},\, \mu_{BC}) = \frac{\mu_{\mathrm{AB}}\mu_{\mathrm{BC}}}{\mu_{\mathrm{AB}} + \mu_{\mathrm{BC}}l_{BC}^2}$$

$$\sigma_k = \sqrt{\left(\left.\frac{\partial g}{\partial k_{AB}}\right|_{\substack{k_{AB}=\mu_{AB}\\ k_{BC}=\mu_{BC}}} \sigma_{AB}\right)^2 + \left(\left.\frac{\partial g}{\partial k_{BC}}\right|_{\substack{k_{AB}=\mu_{AB}\\ k_{BC}=\mu_{BC}}} \sigma_{BC}\right)^2}$$

$$= \sqrt{\left(\frac{\mu_{BC}}{\mu_{BC}l_{BC}^2 + \mu_{AB}} - \frac{\mu_{AB}\mu_{BC}}{\left(\mu_{BC}l_{BC}^2 + \mu_{AB}\right)^2}\right)^2 \sigma_{AB}^2 + \left(\frac{\mu_{AB}}{\mu_{BC}l_{BC}^2 + \mu_{AB}} - \frac{\mu_{AB}\mu_{BC}l_{BC}^2}{\left(\mu_{BC}l_{BC}^2 + \mu_{AB}\right)^2}\right)^2 \sigma_{BC}^2}$$

$$= \frac{\sqrt{\mu_{BC}^4 l_{BC}^4 \sigma_{AB}^2 + \mu_{AB}^4 \sigma_{BC}^2}}{\left(\mu_{BC}l_{BC}^2 + \mu_{AB}\right)^2}$$

Ans.