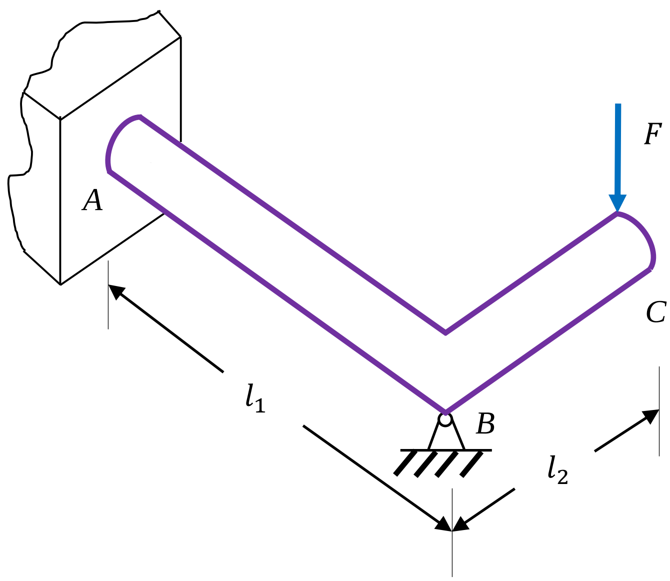
60. As shown in the figure, a torsion bar *AB* fixed at *A* is simply supported at *B* and connected to a cantilever *BC*. A force $F \sim N(1000, 50^2)$ N is applied at *C*. Bar *AB* has a spring rate of $k_1 \sim (2 \times 10^5, (2 \times 10^4)^2)$ N·m/rad and a length of $l_1 \sim N(0.5, 0.001^2)$ m. Cantilever *AB* has a spring rate of $k_2 \sim (3 \times 10^4, (3 \times 10^3)^2)$ N/m. If the allowable deflection at *C* is $\delta_a = 0.05$ m and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the maximum length of *BC* using First Order Second Moment Method. Assume that F, l_1 and l_2 are independent.



Solution

The torque of torsion bar *AB* is

$$M_B = Fl_2$$

Thus the angle of twisting is

$$\theta_B = \frac{M_B}{k_1} = \frac{Fl_2}{k_1}$$

Then the deflection δ_1 at *C* resulted from the twisting of torsion bar *AB* is

$$\delta_1 = \theta_B l_2 = \frac{Fl_2^2}{k_1}$$

For the cantilever *BC*, the deflection δ_2 at *C* resulted from force *F* is

$$\delta_2 = \frac{F}{k_2}$$

Then the overall deflecting at *C* is

$$\delta = \delta_1 + \delta_2 = \frac{F l_2^2}{k_1} + \frac{F}{k_2}$$

The limit-state function is the actual deflection subtracted from the allowable deflection. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - \delta = \delta_a - \frac{F l_2^2}{k_1} - \frac{F}{k_2}$$

where $\mathbf{X}$=(F, k_1, k_2).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \delta_a - \frac{\mu_F l_2^2}{\mu_{k_1}} - \frac{\mu_F}{\mu_{k_2}}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2 + \left(\left.\frac{\partial g}{\partial l_2}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{l_2}\right)^2}$$

$$= \sqrt{\left(\left(-\frac{l_2^2}{\mu_{k_1}} - \frac{1}{\mu_{k_2}}\right)\sigma_F\right)^2 + \left(\frac{\mu_F l_2^2}{\mu_{k_1}^2}\sigma_{k_1}\right)^2 + \left(\frac{\mu_F}{\mu_{k_2}^2}\sigma_{k_2}\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\delta_a - \frac{\mu_F l_2^2}{\mu_{k_1}} - \frac{\mu_F}{\mu_{k_2}}\right)}{\sqrt{\left(\left(-\frac{l_2^2}{\mu_{k_1}} - \frac{1}{\mu_{k_2}}\right)\sigma_F\right)^2 + \left(\frac{\mu_F l_2^2}{\mu_{k_1}^2}\sigma_{k_1}\right)^2 + \left(\frac{\mu_F}{\mu_{k_2}^2}\sigma_{k_2}\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\delta_a - \frac{\mu_F l_2^2}{\mu_{k_1}} - \frac{\mu_F}{\mu_{k_2}}\right)}{\sqrt{\left(\left(-\frac{l_2^2}{\mu_{k_1}} - \frac{1}{\mu_{k_2}}\right)\sigma_F\right)^2 + \left(\frac{\mu_F l_2^2}{\mu_{k_1}^2}\sigma_{k_1}\right)^2 + \left(\frac{\mu_F}{\mu_{k_2}^2}\sigma_{k_2}\right)^2}}$$

$$= \frac{-\left(0.05 - \frac{1000 l_2^2}{2(10^5)} - \frac{1000}{3(10^4)}\right)}{\sqrt{\left(\left(-\frac{l_2^2}{2(10^5)} - \frac{1}{3(10^4)}\right)(50)\right)^2 + \left(\frac{1000 l_2^2}{\left(2(10^5)\right)^2} 2(10^4)\right)^2 + \left(\frac{1000}{\left(3(10^4)\right)^2} 3(10^3)\right)^2}}$$

Solving for l_2 yields

$$l_2 = 0.375 \text{ mm}$$

Ans.