61. A rotating-beam is subjected to a completely reversed stress. The fatigue strength fraction of the beam is $f = 0.8$ and the ultimate strength of the beam is $S_{ut} \sim N(220,\ 20^2)$ kpsi. If the yield strength of the beam is $S_y \sim N(300,\ 30^2)$ kpsi and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the maximum cycles of the reversed stress using the First Order Second Moment Method. Assume that S_{ut} and S_y are independent.

Solution

The failure strength is

$$S_f = S_{ut} N^{(\log f)/3}$$

And thus the completed reversed stress is given by

$$S_{rev} = S_f = S_{ut} N^{(\log f)/3}$$

So the limit-state function is the reversed nominal stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S_{rev} = S_y - S_{ut} N^{(\log f)/3}$$

where $\mathbf{X}$=(S_y, S_{ut}).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_y} - \mu_{S_{ut}} N^{\frac{\log f}{3}}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial S_{ut}}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_{ut}}\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(N^{\frac{\log f}{3}} \sigma_{S_{ut}}\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\mu_{S_y} - \mu_{S_{ut}} N^{\frac{\log f}{3}}\right)}{\sqrt{\left(\sigma_{S_y}\right)^2 + \left(N^{\frac{\log f}{3}} \sigma_{S_{ut}}\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\mu_{S_y} - \mu_{S_{ut}} N^{\frac{\log f}{3}}\right)}{\sqrt{\left(\sigma_{S_y}\right)^2 + \left(N^{\frac{\log f}{3}} \sigma_{S_{ut}}\right)^2}}$$

$$= \frac{-\left(300(10^3) - 220(10^3) N^{\frac{\log(0.8)}{3}}\right)}{\sqrt{(30(10^3))^2 + \left(N^{\frac{\log(0.8)}{3}} 20(10^3)\right)^2}}$$

Solving for N yields

$$N = 84$$

Ans.