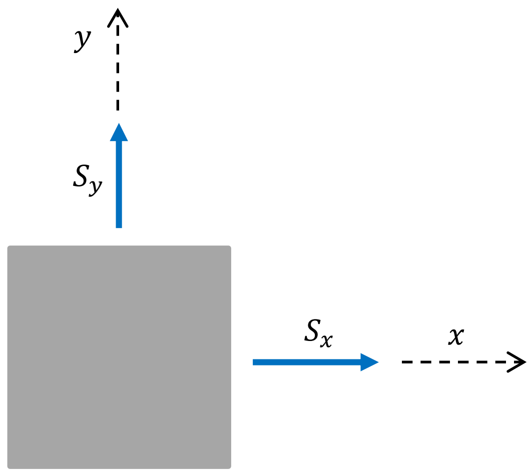
62. A critical stress element is subjected to two normal stresses $S_x \sim N(1,\ 0.1^2)$ MPa and $S_y \sim N(3, 0.3^2)$ MPa. The Poission's ratio is ν is 0.29 and the modulus of elasticity is $E = 90$ MPa. If the axial length of the element is $l_x = 2$ mm and allowable axial elongation is $\delta_a = 0.015$ mm, determine the probability of failure using First Order Second Moment Method. Assume that S_x and S_y are independent.



Solution

The axial strain is

$$\epsilon_x = \frac{1}{E}(S_x - \nu S_y)$$

Therefore the axial elongation is given by

$$\delta_x = \epsilon_x l_x = \frac{1}{E}(S_x - \nu S_y) l_x$$

The limit-state function is the actual axial elongation subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - \delta_x = \delta_a - \frac{1}{E}(S_x - \nu S_y) l_x$$

where $\mathbf{X}=(S_x, S_y)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \delta_a - \frac{1}{E}\left(\mu_{S_x} - \nu \mu_{S_y}\right) l_x$$

$$= 1.5(10^{-5}) - \frac{1}{90(10^6)}\left(1(10^6) - 0.29(3)(10^6)\right)(2)(10^{-3})$$

$$= 1.2111(10^{-3}) \quad \text{m}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_x}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_x}\right)^2 + \left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(\frac{\mu_{l_x}}{E}\sigma_{S_x}\right)^2 + \left(\frac{-\nu\mu_{l_x}}{E}\sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(\frac{2(10^{-3})}{90(10^6)}(1)(10^5)\right)^2 + \left(\frac{-0.29(2)(10^{-3})}{90(10^6)}(3)(10^5)\right)^2}$$

$$= 2.9455(10^{-4}) \ \text{m}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{1.2111(10^{-3})}{2.9455(10^{-4})}\right) = 1.96\ (10^{-5})$$

Ans.