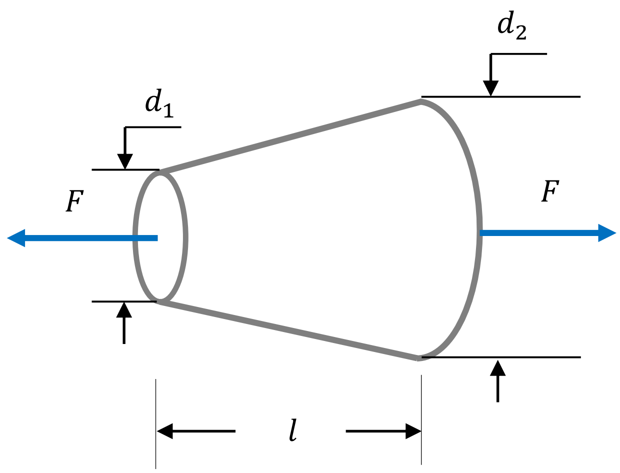
63. A force $F \sim N(3000, 300^2)$ lbf is applied to a tapered bar shown in the figure. The bar has a diameter of $d_1 = 1$ in at one end and a diameter of $d_2 = 2$ in at the other end. The length is $l \sim N(7, 0.07^2)$ in and the modulus of elasticity is $E = 18$ Mpsi. If the allowable axial elongation is $\delta_a = 10^{-3}$ in, determine the probability of failure using the First Order Second Moment Method. Note that the elongation of tapered portion is $\delta = \frac{4}{\pi}\frac{Pl}{d_1 d_2 E}$. And assume that F and l are independent.



Solution

For the tapered portion, the elongation is given by

$$\delta = \frac{4}{\pi}\frac{Fl}{d_1 d_2 E}$$

Thus the limit-state function is the actual elongation subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - \delta = \delta_a - \frac{4}{\pi}\frac{Fl}{d_1 d_2 E}$$

where **X**=(F, l).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \delta_a - \frac{4}{\pi}\frac{\mu_F \mu_l}{d_1 d_2 E} = 10^{-3} - \frac{4}{\pi}\frac{3000(7)}{1(2)(18)(10^6)} = 2.5728(10^{-4}) \text{ in}$$

$$\delta_Y = \sqrt{\left(\frac{\partial g}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial S}{\partial l}\right)^2 \sigma_l^2}$$

$$= \sqrt{\left(-\frac{4}{\pi}\frac{\mu_l}{d_1 d_2 E}\right)^2 \sigma_F^2 + \left(-\frac{4}{\pi}\frac{\mu_l}{d_1 d_2 E}\right)^2 \sigma_{l_2}^2}$$

$$= \sqrt{\left(-\frac{4}{\pi}\frac{7}{1(2)(18)(10^6)}\right)^2 (300)^2 + \left(-\frac{4}{\pi}\frac{3000}{1(2)(18)(10^6)}\right)^2 (0.07)^2}$$

$$= 7.4643(10^{-5}) \text{ in}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{2.5728(10^{-4})}{7.4643(10^{-5})}\right) = 2.84(10^{-4})$$

Ans.