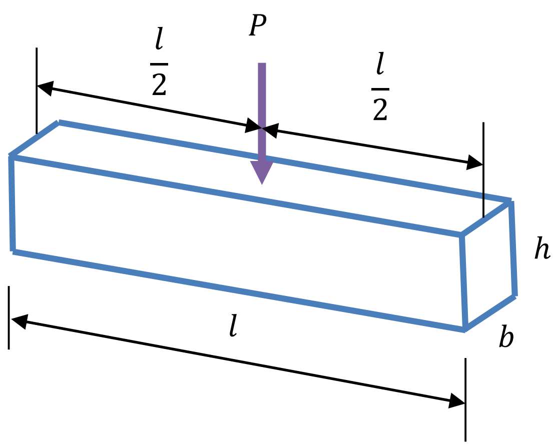
65. A rectangular-cross-section beam is simply-supported and is subjected to a concentrated load of $P \sim N(8000,\ 800^2)$ N as shown in the figure. The width and height of the cross section is $b = 80$ mm and $h = 100$ mm, respectively. If the allowable bending stress is $S_a \sim N(60,\ 6^2)$ MPa and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the maximum length of the beam. Assume that P and S_a are independent.



Solution

The reaction force is given by

$$R_1 = \frac{P}{2}$$

And the maximum bending moment is

$$M = R_1 \frac{l}{2} = \frac{Pl}{4}$$

Thus the bending stress is

$$S = \frac{Mc}{I} = \frac{\frac{Pl}{4}\frac{h}{2}}{\frac{bh^3}{12}} = \frac{3Pl}{2bh^2}$$

The limit-state function is the actual bending stress subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{3Pl}{2bh^2}$$

where $\mathbf{X} = (S_a, P)$

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_a} - \frac{3\mu_P l}{2bh^2}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P\right)^2} = \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{3l}{2bh^2}\sigma_P\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{\mu_{S_a} - \frac{3\mu_P l}{2bh^2}}{\sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{3l}{2bh^2}\sigma_P\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = -\frac{\mu_{S_a} - \frac{3\mu_P l}{2bh^2}}{\sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{3l}{2bh^2}\sigma_P\right)^2}}$$

$$= -\frac{60(10^6) - \frac{3(8000)l}{2\left(80(10^{-3})\right)\left(100(10^{-3})\right)^2}}{\sqrt{(6(10^6))^2 + \left(-\frac{3l}{2\left(80(10^{-3})\right)\left(100(10^{-3})\right)^2}(800)\right)^2}}$$

Solving for l yields

$$l = 2.08 \text{ m}$$

Ans.