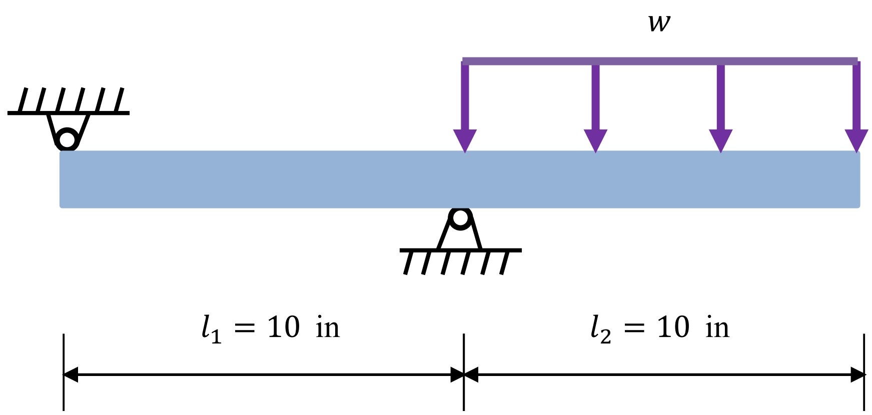
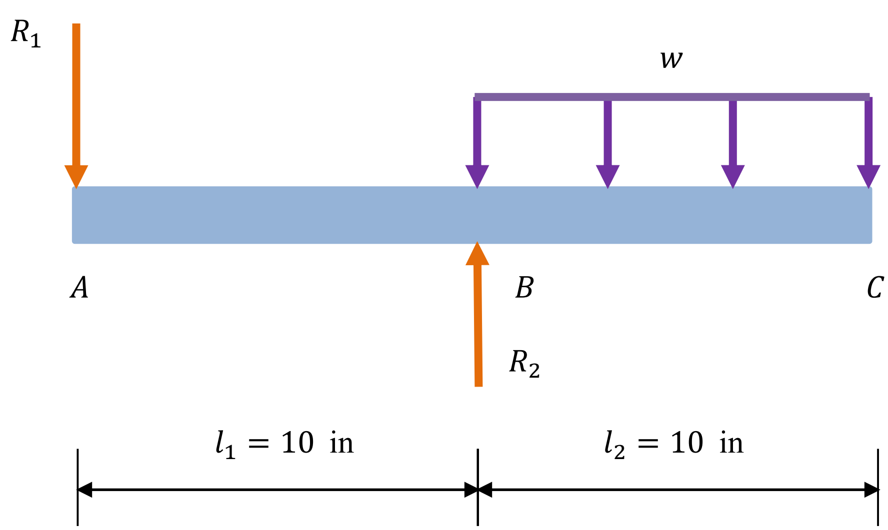
66. A beam with a round cross section is subjected to a uniform load of $w \sim N(200,\ 20^2)$ lbf/in. The diameter of the beam is $d = 2$ in. If the allowable bending stress is $S_a \sim N(22,\ 2^2)$ kpsi, estimate the probability of failure using the First Order Second Moment Method. Assume that w and S_a are independent.



Solution

Consider the force equilibrium of beam shown below.



According to the moment equilibrium of beam with respect to B,

$$R_1 l_1 = w l_2 \frac{l_2}{2}$$

The maximum bending moment occurs at point *B* and it is given by

$$M = R_1 l_1 = w l_2 \frac{l_2}{2} = \frac{l_2^2}{2} w$$

Thus the maximum bending stress is given by

$$S = \frac{Mc}{I} = \frac{M \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32 \frac{l_2^2}{2} w}{\pi d^3} = \frac{16 l_2^2}{\pi d^3} w$$

The limit-state function is the maximum bending stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{16 l_2^2}{\pi d^3} w$$

where $\mathbf{X}=(S_a, w)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_a} - \frac{16 l_2^2}{\pi d^3} \mu_w = 22(10^3) - \frac{16(10^2)}{\pi 2^3}(200) = 9.2676(10^3) \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial w}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_w\right)^2}$$

$$= \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{16 l_2^2}{\pi d^3} \sigma_w\right)^2}$$

$$= \sqrt{\left(2(10^3)\right)^2 + \left(-\frac{16(10^2)}{\pi 2^3}(20)\right)^2}$$

$$= 2.3709(10^3) \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-9.2676(10^3)}{2.3709(10^3)}\right) = 4.64(10^{-5})$$ **Ans.**