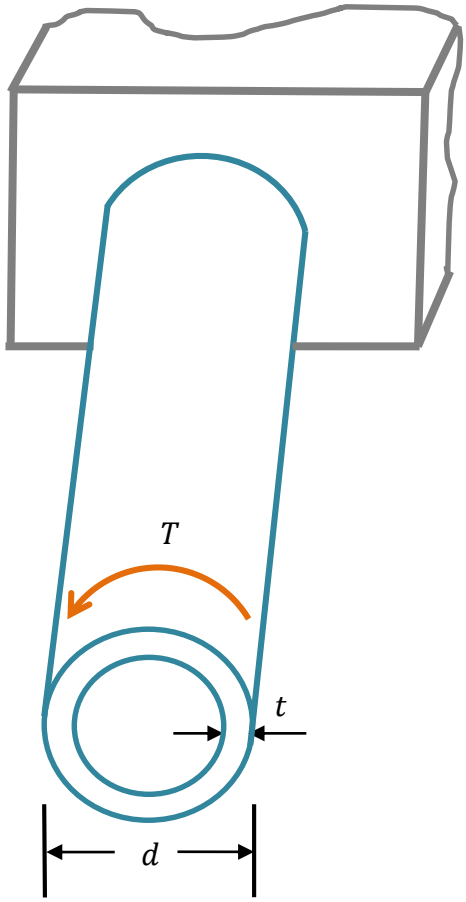
69. A welded steel tube is in torsion . The tube is thin-wall and is of round section with a diameter of $d = 40$ mm. It has a a thickness of $t = 8$ mm and a length of $l \sim N(800, 0.1^2)$ mm. The shear modulus is $G = 80$ GPa. If the allowable shear stress is $\tau_a \sim N(50, 5^2)$ MPa, estimate the mean and standard deviation of the angle of twist using the First Order Second Moment Method. Note that l and τ_a are independent.



Solution

According to the theory of closed thin-walled tube, the shear stress is

$$\tau = \frac{T}{2A_m t}$$

Where A_m is the area enclosed by section median line and is given by

$$A_m = \frac{\pi}{4}(d - t)^2$$

The allowable torque is given by

$$T = 2\tau_a A_m t$$

Thus the angular twist per unit of length is

$$\theta_1 = \frac{TL_m}{4GA_m^2 t} = \frac{2\tau_a A_m t L_m}{4GA_m^2 t} = \frac{\tau_a}{2G}\frac{L_m}{A_m}$$

where $L_m = \pi(d-t)$ is the section median line.

So the angle of twist can be obtained by

$$\theta = \theta_1 l = \frac{\tau_a}{2G}\frac{L_m}{A_m} l = \frac{\tau_a}{2G}\frac{\pi(d-t)}{\frac{\pi}{4}(d-t)^2} l = \frac{2}{G(d-t)}\tau_a l$$

Let

$$Y = g(\mathbf{X}) = \theta = \frac{2}{G(d-t)}\tau_a l$$

where $\mathbf{X}=(\tau_a, l)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \frac{2}{G(d-t)}\mu_{\tau_a}\mu_l$$

$$= \frac{2}{(80)(10^9)\left(40(10^{-3}) - 5(10^{-3})\right)} 50(10^6)(800)(10^{-3})$$

$$= 3.13(10^{-2})$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\Big|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\frac{\partial g}{\partial l}\Big|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_l\right)^2}$$

$$= \sqrt{\left(\frac{2\mu_l}{G(d-t)}\sigma_{\tau_a}\right)^2 + \left(\frac{2\mu_{\tau_a}}{G(d-t)}\sigma_l\right)^2}$$

$$= \sqrt{\begin{matrix}\left(\frac{2(800)(10^{-3})}{(80)(10^9)\left(40(10^{-3}) - 5(10^{-3})\right)} 5(10^6)\right)^2 \\ +\left(\frac{2(50)(10^6)}{(80)(10^9)\left(40(10^{-3}) - 5(10^{-3})\right)} (0.1)(10^{-3})\right)^2\end{matrix}}$$

$$= 3.13(10^{-3})$$

Ans.