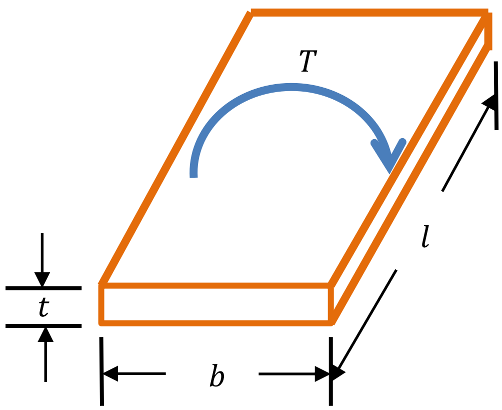
70. A 500-mm-long strip is subjected to a torque $T \sim N(30, 3^2)$ N·m shown in the figure. The strip has a rectangular cross section with a width of $b = 150$ mm and a thickness of $t = 40$ mm. If the allowable shear stress is $\tau_a \sim N(80, 10^2)$ MPa, estimate probability of failure using the First Order Second Moment Method. Note that T and τ_a are independent.



Solution

For a rectangular section strip in torsion, the maximum shearing stress is given by

$$\tau \cong \frac{T}{bt^2}(3 + \frac{1.8}{b/t})$$

Thus the limit-state function is the maximum shearing stress subtracted from allowable one. Failure occurs when $Y < 0$

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{1}{bt^2}(3 + \frac{1.8}{b/t})T$$

where $\mathbf{X}=(\tau_a, T)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{\tau_a} - \frac{1}{bt^2}\left(3 + \frac{1.8}{b/t}\right)\mu_T$$

$$= 80(10^6) - \frac{1}{150(10^{-3})\left(40(10^{-3})\right)^2}(3 + \frac{1.8}{150(10^{-3})/40(10^{-3})})30$$

$$= 4.2125(10^7) \text{ Pa}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial \tau_a}\right|_{\mu_X} \sigma_{\tau_a}\right)^2 + \left(\left.\frac{\partial g}{\partial T}\right|_{\mu_X} \sigma_T\right)}$$

$$= \sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{1}{bt^2}(3+\frac{1.8}{b/t})\sigma_T\right)^2}$$

$$= \sqrt{\left(10(10^6)\right)^2 + \left(-\frac{1}{150(10^{-3})\left(40(10^{-3})\right)^2}(3+\frac{1.8}{150(10^{-3})/40(10^{-3})})3\right)^2}$$

$$= 1.0693(10^7) \quad \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{4.2125(10^7)}{1.0693(10^7)}\right) = 4.08(10^{-5})$$

Ans.