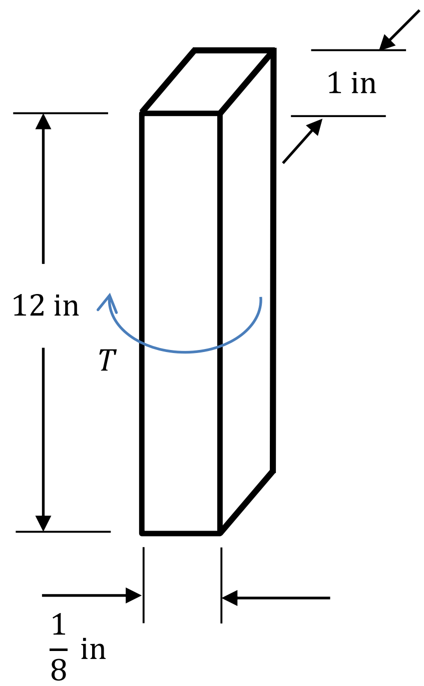
8. A strip of steel is subjected to a torque $T \sim N(60, 1^2)$ lbf·in. The steel is 12 in long, $\frac{1}{8}$ in thick, and 1 in wide. The allowable shear stress of the steel is $\tau_a \sim (14, 0.5^2)$ kpsi. Determine the probability of failure of the strip using the First Order Second Moment Method.



Solution

The length of the median line is 1 in. Based on the membrane analogy theory,

$$\tau = G\theta_1 c = \frac{3T}{Lc^2}$$

where τ is the shear stress, G is the shear modulus, θ_1 is the angle of twist per unit length, T is torque, L is the length of the median line, and c is the wall thickness.

The limit-state function is the shear stress of the strip subtracted from the allowable shear stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{3T}{Lc^2} = \tau_a - \frac{3T}{1\left(\frac{1}{8}\right)^2} = \tau_a - 192T$$

where $\mathbf{X}=(\tau_a, T)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - 192\mu_T = 14000 - 11520 = 2480 \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial \tau_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\left.\frac{\partial g}{\partial T}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2} = \sqrt{\left(\sigma_{\tau_a}\right)^2 + (-192\sigma_T)^2}$$

$$= \sqrt{(500)^2 + (-192(1))^2} = 535.60 \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-2480}{535.60}\right) = 1.8255(10^{-6}) \qquad \textbf{Ans.}$$