9. A cylindrical pressure vessel made of tubing carries an internal pressure $p \sim N(300, 40^2)$ psi. It has an outside diameter of $d_o = 10$ in and a wall thickness $t = \frac{1}{5}$ in. If the allowable tangential stress of the vessel is $S_a \sim N(12, 1^2)$ kpsi, determine the probability of failure using the First Order Second Moment Method. Use the theory of thin-walled vessels.

Solution

The inside diameter of the vessel is

$$d_i = d_o - 2t = 10 - 2(0.2) = 9.6 \text{ in}$$

Then

$$\frac{t}{r_i} = \frac{t}{d_i/2} = \frac{0.2}{9.6/2} = 0.0417 < 0.1$$

Applying the theory of thin-walled vessels, we have the maximum tangential stress as follows:

$$S_{max} = \frac{p(d_i + t)}{2t}$$

The limit-state function is the maximum tangential stress of the vessel subtracted from the allowable tangential stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S_{max} = S_a - \frac{p(d_i + t)}{2t} = S_a - 24.5p$$

where $\mathbf{X}=(S_a, p)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_a} - 24.5\mu_p = 12000 - 24.5(300) = 4650 \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_a}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_a}\right)^2 + \left(\frac{\partial g}{\partial p}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_p\right)^2} = \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-24.5\sigma_p\right)^2}$$

$$= \sqrt{(1000)^2 + (-24.5(40))^2} = 1400.14 \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-4650}{1400.14}\right) = 4.4833(10^{-4})$$ **Ans.**