13. A solid shaft with a diameter of $d \sim N(50, 0.1^2)$ mm is designed to tranismit a torque. The design team considers to replace the solid shaft with a hollow shaft having an outside diameter of $d_o \sim N(50, 0.1^2)$ mm and an inside diameter of $d_i \sim N(36, 0.1^2)$ mm. If the two shafts have the same length and density, determine the mean and standard deviation of the percentage reduction in the shaft weight using FOSM. Note that d, d_o and $d_i$are independent.

Solution

The weight of the solid shaft is

$$W_{solid} = \frac{\pi}{4} d^2 L\rho$$

where L is the length of the shaft, and ρ is the density.

The weight of the hollow shaft is

$$W_{hollow} = \frac{\pi}{4}(d_o^2 - d_i^2)L\rho$$

So the percentage reduction in shaft weight is

$$\Delta W = \frac{W_{solid} - W_{hollow}}{W_{solid}}(100\%) = \frac{d^2 - (d_o^2 - d_i^2)}{d^2} = 1 - \frac{d_o^2}{d^2} + \frac{d_i^2}{d^2}$$

Let

$$Y = g(\mathbf{X}) = 1 - \frac{d_o^2}{d^2} + \frac{d_i^2}{d^2}$$

where **X**=(d, d_o, d_i).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = 1 - \frac{\mu_{d_o}^2}{\mu_d^2} + \frac{\mu_{d_i}^2}{\mu_d^2} = 1 - \frac{50}{50} + \frac{36}{50} = 51.84\%$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_d\right)^2 + \left(\frac{\partial g}{\partial d_o}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{d_o}\right)^2 + \left(\frac{\partial g}{\partial d_i}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{d_i}\right)^2}$$

$$= \sqrt{\left(\left(2\frac{\mu_{d_o}^2}{\mu_d^3} - 2\frac{\mu_{d_i}^2}{\mu_i^3}\right)\sigma_d\right)^2 + \left(\left(-2\frac{\mu_{d_o}}{\mu_d^2}\right)\sigma_{d_o}\right)^2 + \left(2\frac{\mu_{d_i}}{\mu_d^2}\sigma_{d_i}\right)^2}$$

$$= \sqrt{\left(\left(2\frac{50^2}{50^3} - 2\frac{36^2}{50^3}\right)(0.1)\right)^2 + \left(\left(-2\frac{50}{50^2}\right)(0.1)\right)^2 + \left(2\frac{36}{50^2}(0.1)\right)^2}$$

$= 0.53\%$

Ans.