14. A hollow steel shaft is subjected to a torque $T \sim N(4200, 400^2)$ N·m. The allowabe torsional stress is $\tau_a \sim N(120, 10^2)$ MPa. If the inside diameter is designed to be 80% of the outiside diameter and the probability of failure is designed to be $p_f = 1 \times 10^{-5}$, determine the size of shaft and choose a preferred one using FOSM. Note that T and τ_a are independent.

Solution

For a shaft in torsion, the maximum torsional stress is given by

$$\tau = \frac{Tr}{J} = \frac{T\frac{d}{2}}{\frac{\pi}{32}(d^4 - (0.8d)^4)} = \frac{16T}{0.5904\pi d^3}$$

Thus the limit-state function is the maximum torsional stress subtracted from the allowable one. Failure occurs when $Y < 0$

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{16T}{0.5904\pi d^3}$$

where **X**=(τ_a, T).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - \frac{16\mu_T}{0.5904\pi d^3}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\frac{\partial g}{\partial T}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2}$$

$$= \sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{16}{0.5904\pi d^3}\sigma_T\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{\mu_{\tau_a} - \frac{16\mu_T}{0.5904\pi d^3}}{\sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{16}{0.5904\pi d^3}\sigma_T\right)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = -\frac{\mu_{\tau_a} - \frac{16\mu_T}{0.5904\pi d^3}}{\sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{16}{0.5904\pi d^3}\sigma_T\right)^2}}$$

$$= -\frac{120(10^6) - \frac{16(4200)}{0.5904\pi d^3}}{\sqrt{(10(10^6))^2 + \left(-\frac{16}{0.5904\pi d^3}(400)\right)^2}}$$

Solving for d yields

$$d = 80.69 \text{ mm}$$

Thus $d = 100$ mm can be used.

Ans.