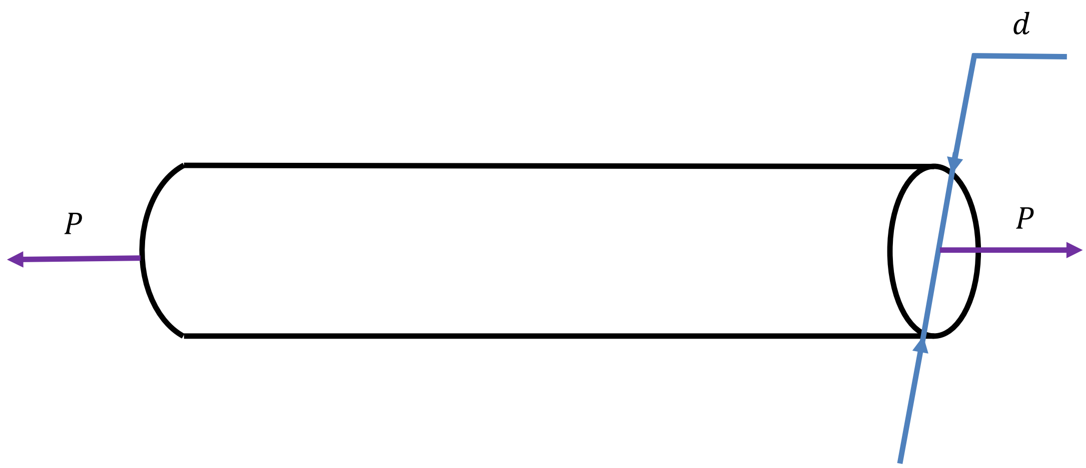
2. A solid circular rod is subjected to an axial force of $P = 2000$ lbf. The yield stress of the rod is $S_y = 20$ kpsi. The factor of safety is $n_s = 3.0$. a) What is the minimum diameter of the rod? Then select a preferred fractional diameter. b) If $P \sim N(2000, 300^2)$ lbf, $S_y \sim N(20, 3^2)$ kpsi, and P and S are independent, determine the probability of failure using the First Order Moment Method.



Solution

a) The cross-sectional area of the rod is

$$A = \frac{\pi}{4} d^2$$

The tensile stress of the rod is

$$S = \frac{P}{A} = \frac{4P}{\pi d^2}$$

The tensile stress should be less than the allowable stress

$$S = \frac{4P}{\pi d^2} \leq \frac{S_y}{n_s}$$

Solving for d yields

$$d \geq \sqrt{\frac{4Pn_s}{\pi S_y}} = \sqrt{\frac{4(2000)(3)}{\pi(20000)}} = 0.618 \text{ in}$$

Thus the minimum diameter of the rod is 0.618 in. **Ans.**

And the preferred fractional diameter could be chosen as $d_0 = \frac{5}{8}$ in = 0.625 in. **Ans.**

b) The limit-state function is the actual stress of the rod subtracted from the allowable maximum stress.

Failure occurs when $Y < 0$

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{4P}{\pi d_0^2}$$

where $\mathbf{X} = (S_y,\ P)$, and d_0= 0.625 in is the preferred diameter.

Using FOSM, we have

$$\mu_g = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{4\mu_P}{\pi d_0^2} = 20000 - \frac{4(2000)}{\pi(0.625^2)} = 1.3481(10^4)\ \text{psi}$$

$$\sigma_g = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2} = \sqrt{\left(\sigma_{S_y}\right)^2 + \left(\frac{4}{\pi d_0^2}\sigma_P\right)^2} = \sqrt{3000^2 + \left(\frac{4}{\pi(0.625^2)}300\right)^2}$$

$$= 3.1553(10^3)\ \text{psi}$$

Evaluate the probability of failure

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-13481}{3155.3}\right) = 9.6673(10^{-6})$$ **Ans.**