4. The allowable shear stress of a shaft is $\tau_a = 80$ MPa, and the shaft speed is $n = 2500$ rev/min. a) What is the minimum diameter of the shaft to transmit 50 KW? Then select a preferred diameter. b) If $\tau_a \sim N(80, 3^2)$ MPa and $n \sim N(2500, 100^2)$ rev/min and τ_a and n are independent, determine the probability of failure using the First Order Second Moment Method.

Solution

a)

The torque can be obtained from the given power and speed.

$$T = 9.55\frac{H}{n} = 9.55\frac{(50000)}{2500} = 191 \ \ \text{N·m}$$

where H is the power, and n is the shaft speed.

The maximum shear stress developing throughout the cross section is

$$\tau_{max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

where r is the radius of the shaft, d is the diameter, and J is the polar second moment of area.

The maximum shear stress should be less than the allowable shear stress.

$$\tau_{max} = \frac{16T}{\pi d^3} \leq \tau_a$$

Solving for d yields

$$d \geq \left(\frac{16T}{\pi \tau_a}\right)^{\frac{1}{3}} = \left(\frac{16(191)}{\pi(80)(10^6)}\right)^{\frac{1}{3}} = 0.0230 \ \text{m} = 23.0 \text{ mm}$$

Thus the minimum diameter of the shaft is 23.0 mm. **Ans.**

And the preferred diameter could be chosen as $d_0 = 25$ mm **Ans.**

b)

The limit-state function is the actual maximum shear stress of the shaft subtracted from the allowable maximum shear stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau_{max} = \tau_a - \frac{16T}{\pi d_0^3} = \tau_a - \frac{152.8H}{\pi d_0^3 n}$$

where **X**$=(\tau_a, n)$, and d_0= 25 mm is the preferred diameter.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{\tau_a} - \frac{152.8H}{\pi d_0^3 \mu_n} = 80000000 - \frac{152.8(50000)}{\pi(0.025^3)2500} = 1.7744(10^7) \quad \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial \tau_a}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{\tau_a}\right)^2 + \left(\left.\frac{\partial g}{\partial n}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_n\right)^2} = \sqrt{(\sigma_{\tau_a})^2 + \left(-\frac{152.8H}{\pi d_0^3 \mu_n^2}\sigma_n\right)^2}$$

$$= \sqrt{(3(10^6))^2 + \left(\frac{152.8(50000)}{\pi 0.025^3(2500^2)}100\right)^2} = 3.8989(10^6) \quad \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-1.7744(10^7)}{3.8989(10^6)}\right) = 2.6702(10^{-6})$$ **Ans.**