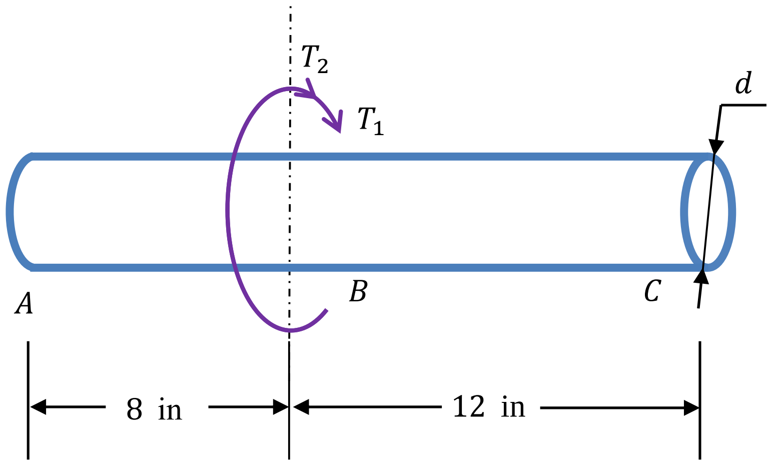
5. A shaft is fixed at point *A* and *C*. It is subjected to two toques acting at point *B*. The diamater of the shaft is $d = 1$ in. If $T_1 \sim N(120,\ 10^2)$ lbf·in, $T_2 \sim N(80,\ 8^2)$ lbf·in, and T_1 and T_2 are independent, determine the distribution of the angel of twist at *B*.



Solution

Since the angle of twist $\theta_{AB} = \theta_{BC}$,

$$\frac{T_A l_{AB}}{JG} = \frac{T_C l_{BC}}{JG}$$

where T_A and T_c are the torque reactions at *A* and *C*, respectively, J is the torsional constant, and G is the shear modulus of steel.

Then

$$T_A = \frac{l_{BC}}{l_{AB}} T_C = \frac{3}{2} T_C$$

According to the moment equilibrium of shaft *AC*

$$T_A + T_C = T_1 + T_2$$

Then

$$T_C = \frac{2}{5}(T_1 + T_2)$$

Thus, the angle of twist at *B* is

$$\theta_B = \frac{T_C l_{BC}}{JG} = \frac{\frac{2}{5}(T_1 + T_2)(12)}{\frac{\pi}{32}(1)^4(11.5)10^6} = 4.25(10^{-6})(T_1 + T_2)$$

Since T_1 and T_2 are independently and normally distributed, their linear combination, θ_B , is also normally distributed. The mean and standard deviation of θ_B are given by

$$\mu_{\theta_B} = 4.25(10^{-6})\left(\mu_{T_1} + \mu_{T_2}\right) = 4.25(10^{-6})(120 + 80) = 8.50(10^{-4}) \ \text{rad}$$

$$\sigma_{\theta_B} = \sqrt{\left(4.25(10^{-6})\right)^2 \sigma_{T_1}^2 + \left(4.25(10^{-6})\right)^2 \sigma_{T_2}^2} = 4.25(10^{-6})\sqrt{(10^2 + 8^2)} = 5.44(10^{-5}) \ \text{rad}$$

So the distribution of the angel of twist at B is $\theta_B \sim N\left(8.50(10^{-4}),\ \left(5.44(10^{-5})\right)^2\right)$ rad **Ans.**