6. A circular shaft is subjected to an axial force $P{\sim}N(30, 3^2)$ kN. The tensile spring constant of the shaft is $k{\sim}N(2\times 10^9,\ (2\times 10^7)^2)$ N/m. If the allowable axial extension is $\delta_a = 0.02$ mm, estimate the probability of failure using the First Order Second Moment Method. Assume P and k are independent.

Solution

The extension of the shaft is

$$\delta = \frac{P}{k}$$

Thus the limit-state function is the actual extension subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \delta_a - \delta = 0.02(10^{-3}) - \frac{P}{k}$$

where **X**=(P, k).

We can rewrite the limit-state function as

$$Y^* = 0.02(10^{-3})k - P$$

Because k and P are independently and normally distributed, their linear combination Y^* is also normally distributed and

$$\mu_{Y^*} = 0.02(10^{-3})\mu_k - \mu_P = 0.02(10^{-3})2(10^9) - 30(10^3) = 1(10^4)\ \text{N}$$

$$\begin{aligned}\sigma_{Y^*} &= \sqrt{(0.02(10^{-3})\sigma_k)^2 + (\sigma_P)^2}\\ &= \sqrt{\left(0.02(10^{-3})(2)(10^7)\right)^2 + \left(3(10^3)\right)^2}\\ &= 3.03(10^3)\ \text{N}\end{aligned}$$

The probability of failure is then given by

$$p_f = \Pr(Y^* < 0) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1(10^4)}{3.03(10^3)}\right) = 4.76(10^{-4})$$ **Ans.**