8. A shaft is designed to transmit a power $H = 8$ kW at 3000 rpm. The shaft has a round cross section with a diameter of $d \sim N(50, 0.5^2)$ mm . If the allowable shear stress of the shaft is $\tau_a \sim N(100, 10^2)$ MPa, estimate the probability of failure using the First Order Second Moment Method.

Solution

The transmitted power is

$$H = \frac{Tn}{9.55}$$

Thus the required torque is

$$T_r = \frac{9.55H}{n}$$

And the maximum shear stress resulting from the torque is

$$\tau_{\max} = \frac{16T_r}{\pi d^3} = \frac{152.8H}{\pi d^3 n}$$

The limit-state function is the actual maximum shear stress subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau_{\max} = \tau_a - \frac{152.8H}{\pi d^3 n}$$

where $\mathbf{X}=(\tau_a, d)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{\tau_a} - \frac{152.8H}{\pi \mu_d^3 n} = 100(10^6) - \frac{152.8(8)(10^3)}{\frac{\pi\left(50(10^{-3})\right)^3 3000}{60}} = 3.7744(10^7)\ \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\right)^2 \sigma_{\tau_a}^2 + \left(\frac{\partial g}{\partial d}\right)^2 \sigma_d^2}$$

$$= \sqrt{\sigma_{\tau_a}^2 + \left(\frac{152.8H}{\pi \mu_d^4 n}(-3)\right)^2 \sigma_d^2}$$

$$= \sqrt{\left(10(10^6)\right)^2 + \left(\frac{152.8(8)(10^3)}{\frac{\pi\left(50(10^{-3})\right)^4 3000}{60}}(-3)\right)^2 \left((0.5)(10^{-3})\right)^2}$$

$$= 1.0173(10^7)\ \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-3.7744(10^7)}{1.0173(10^7)}\right) = 1.04(10^{-4})$$ **Ans.**