9. A shaft has a circular cross section with a diameter of $d \sim N(80, 0.8^2)$ mm and is designed to transmit a power $H = 1$ kW. If the allowable shear stress of the shaft is $\tau_a \sim N(120, 12^2)$ MPa and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum shaft working speed to transmit the required power.

Solution

The transmitted power is given by

$$H = \frac{Tn}{9.55}$$

Thus the required torque is

$$T_r = \frac{9.55H}{n}$$

And the maximum shear stress resulting from the torque is

$$\tau_{\max} = \frac{16T_r}{\pi d^3} = \frac{152.8H}{\pi d^3 n}$$

The limit-state function is the actual maximum shear stress subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau_{\max} = \tau_a - \frac{152.8H}{\pi d^3 n}$$

where $\mathbf{X}=(\tau_a, d)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{\tau_a} - \frac{152.8H}{\pi \mu_d^3 n}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\right)^2 \sigma_{\tau_a}^2 + \left(\frac{\partial g}{\partial d}\right)^2 \sigma_d^2}$$

$$= \sqrt{\sigma_{\tau_a}^2 + \left(\frac{152.8H}{\pi \mu_d^4 n}(-3)\right)^2 \sigma_d^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\mu_{\tau_a} - \frac{152.8H}{\pi\mu_d^3 n}\right)}{\sqrt{\sigma_{\tau_a}^2 + \left(\frac{152.8H}{\pi\mu_d^4 n}(-3)\right)^2 \sigma_d^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\mu_{\tau_a} - \frac{152.8H}{\pi\mu_d^3 n}\right)}{\sqrt{\sigma_{\tau_a}^2 + \left(\frac{152.8H}{\pi\mu_d^4 n}(-3)\right)^2 \sigma_d^2}}$$

$$= \frac{-\left(120(10^6) - \frac{152.8(1)(10^3)}{\pi\left(80(10^{-3})\right)^3 n}\right)}{\sqrt{\left(12(10^6)\right)^2 + \left(\frac{152.8(1)(10^3)}{\pi\left(80(10^{-3})\right)^4 n}(-3)\right)^2 \left((0.8)(10^{-3})\right)^2}}$$

Solving for n yields

$$n = 5023 \text{ rpm}$$

Ans.