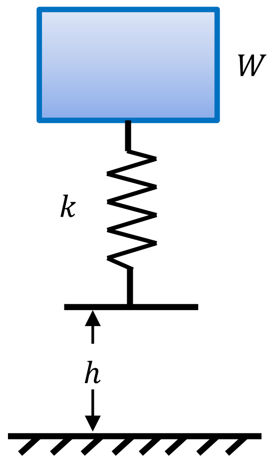
1. An impact system has a weight $W \sim N(60,\ 6^2)$ lbf and a spring constant $k \sim N(200,\ 20^2)$ lbf/in, where W and k are independent. If the system is dropped from $h = 2$ in, determine the mean and standard deviation of the maximum spring force using the First Order Second Moment Method. Ignore the mass of the spring.



Solution

Let the maximum deflection of the spring be δ. According to conservation of energy

$$0 = \frac{1}{2}k\delta^2 - W(h + \delta)$$

Then

$$\delta^2 - \frac{2W}{k}\delta - \frac{2W}{k}h = 0$$

Solving for δ yields

$$\delta = \frac{W}{k} \pm \frac{W}{k}\sqrt{1 + \frac{2hk}{W}}$$

By taking the positive root, the maximum deflection of the spring is

$$\delta_{max} = \frac{W}{k} + \frac{W}{k}\sqrt{1 + \frac{2hk}{W}}$$

Then the maximum spring force is given by

$$F_{max} = k\delta_{max} = W + W\sqrt{1 + \frac{2hk}{W}} = W + \sqrt{W^2 + 2hkW}$$

Using FOSM, we have

$$\mu_{F_{max}} = \mu_W + \sqrt{\mu_W^2 + 2h\mu_k\mu_W} = 60 + \sqrt{60^2 + 2(2)(200)60} = 287.16 \quad \text{lbf}$$

$$\sigma_{F_{max}} = \sqrt{\left(\left.\frac{\partial F_{max}}{\partial W}\right|_{\substack{W=\mu_W \\ k=\mu_k}} \sigma_W\right)^2 + \left(\left.\frac{\partial F_{max}}{\partial k}\right|_{\substack{W=\mu_W \\ k=\mu_k}} \sigma_k\right)^2}$$

$$= \sqrt{\left(\left(1 + \frac{\mu_W + h\mu_k}{\sqrt{\mu_W^2 + 2h\mu_k\mu_W}}\right)\sigma_W\right)^2 + \left(\left(\frac{h\mu_W}{\sqrt{\mu_W^2 + 2h\mu_k\mu_W}}\right)\sigma_k\right)^2}$$

$$= \sqrt{\left(\left(1 + \frac{60 + 2(200)}{\sqrt{60^2 + 2(2)(200)(60)}}\right)6\right)^2 + \left(\left(\frac{2(60)}{\sqrt{60^2 + 2(2)(200)(60)}}\right)20\right)^2}$$

$$= 21.00 \quad \text{lbf}$$

Ans.