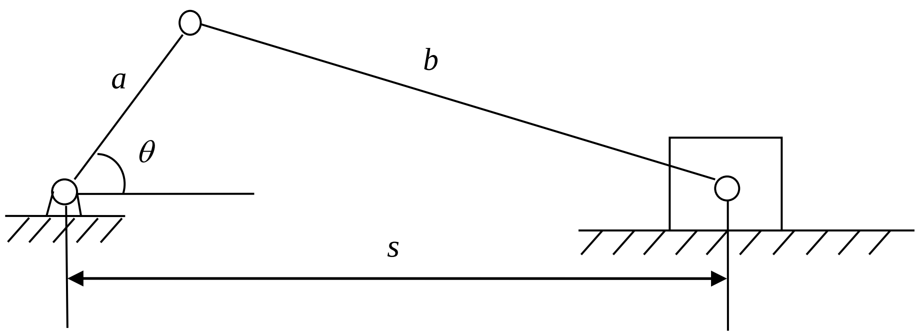
Example 2

The position of a slider-crank mechanism is required to be $s_a = 49.8$ mm when $\theta = 50^\circ$. If the actual position s is outside the tolerance range $s_a \pm 0.6$ mm, it is considered that a failure occurs. The random variables are independent and follow normal distributions of $a \sim N(20, 0.1^2)\,\text{mm}$, $b \sim N(40, 0.12^2)\,\text{mm}$. Use the First Order Second Moment (FOSM) method to calculate the probability of failure.



Solution:

The vector of random variables is $\mathbf{X} = (a, b)$. The actual position of the slider is given by

$$s = a\cos\theta + \sqrt{b^2 - (a\sin\theta)^2}$$

Since the tolerance range is $s_a \pm 0.6$, failure occurs when $s > s_a + 0.6$ or $s < s_a - 0.6$. Let $Y_1 = g_1(\mathbf{X}) = s_a + 0.6 - s$, and $Y_2 = g_2(\mathbf{X}) = s - (s_a - 0.6)$. Then, the probability of failure of the mechanism is given by

$$p_f = \Pr(Y_1 < 0) + \Pr(Y_2 < 0)$$

Since the random variables are independent and follow normal distributions, according to the FOSM method, the mean and standard deviation of Y_1 are calculated by

$$\mu_{Y_1} = g_1(\boldsymbol{\mu}_{\mathbf{X}}) = (s_a + 0.6) - \left(\mu_{\mathrm{a}} \cos\theta + \sqrt{(\mu_b)^2 - (\mu_{\mathrm{a}} \sin\theta)^2}\right) = 0.5947 \text{ mm}$$

$$\frac{\partial g_1}{\partial a} = -\left(\cos\theta + \frac{1}{2}\left(\frac{1}{\sqrt{b^2 - (a\sin\theta)^2}}\right)\left(-2a\sin\theta\right)(\sin\theta)\right)\Bigg|_{(b=\mu_b,\ a=\mu_a)} = -0.3252$$

$$\frac{\partial g_1}{\partial b} = -\left(\frac{1}{2}\left(\frac{1}{\sqrt{b^2 - (a\sin\theta)^2}}\right)(2b)\right)\Bigg|_{(b=\mu_b,\ a=\mu_a)} = -1.0826$$

$$\sigma_{Y_1} = \sqrt{\left(\frac{\partial g_1}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial g_1}{\partial b}\right)^2 \sigma_b^2}$$
$$= \sqrt{\left(-0.3252\right)^2 (0.1)^2 + \left(-1.0862\right)^2 \left(0.12\right)^2} = 0.1339 \text{ mm}$$

Thus, we have

$$\Pr(Y_1 < 0) = \Phi\left(-\frac{\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-4.4408) = 4.4818 \times 10^{-6}$$

Similarly, the mean and standard deviation of Y_2 are calculated by

$$\mu_{Y_2} = g_2(\boldsymbol{\mu}_{\mathbf{X}}) = \left(\mu_{\mathrm{a}} \cos\theta + \sqrt{(\mu_b)^2 - (\mu_{\mathrm{a}} \sin\theta)^2}\right) - (s_a - 0.1) = 0.6053 \text{ mm}$$

$$\frac{\partial g_2}{\partial a} = \left(\cos\theta + \frac{1}{2}\left(\frac{1}{\sqrt{b^2 - (a\sin\theta)^2}}\right)\left(-2a\sin\theta\right)(\sin\theta)\right)\Bigg|_{(b=\mu_b,\, a=\mu_a)} = 0.3252$$

$$\frac{\partial g_2}{\partial b} = \left(\frac{1}{2}\left(\frac{1}{\sqrt{b^2 - (a\sin\theta)^2}}\right)(2b)\right)\Bigg|_{(b=\mu_b,\, a=\mu_a)} = 1.0826$$

$$\sigma_{Y_2} = \sqrt{\left(\frac{\partial g_2}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial g_2}{\partial b}\right)^2 \sigma_b^2}$$
$$= \sqrt{\left(0.3252\right)^2 (0.1)^2 + \left(1.0826\right)^2 \left(0.12\right)^2} = 0.1339 \text{ mm}$$

We have

$$\Pr(Y_2 < 0) = \Phi\left(-\frac{\mu_{Y_2}}{\sigma_{Y_2}}\right) = \Phi(-4.5202) = 3.0893\times10^{-6}$$

The probability of failure of this mechanism is then given by

$$p_f = \Pr(Y_1 < 0) + \Pr(Y_2 < 0) = 4.4818\times10^{-6} + 3.0893\times10^{-6} = 7.5711\times10^{-6}$$ **Ans.**