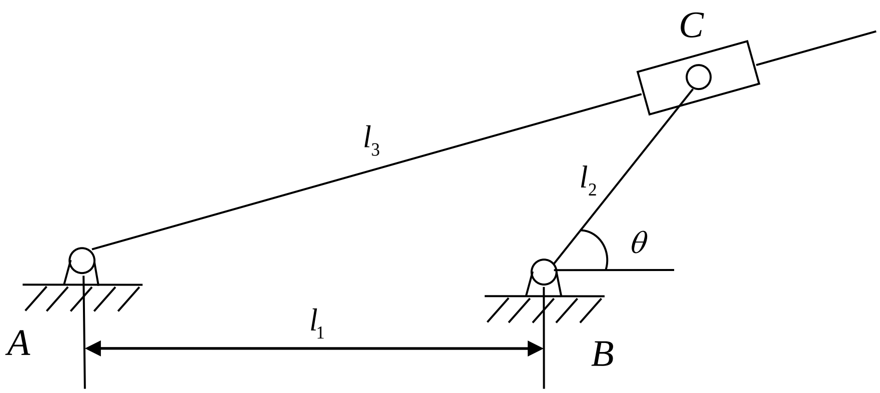
Example 3

The position of a slider-block linkage is required to be $l_{3a} = 6.08$ m when $\theta = 60^\circ$. A failure occurs if the actual position l_3 is outside the tolerance range $l_{3a} \pm 0.027$ m. The design variables are independent and follow normal distributions of $l_1 \sim N(4, 0.002^2)$ m, $l_2 \sim N(3, 0.001^2)$ m. Since uncertainty exists in the measurement of θ, the angle also follows a normal distribution with $\theta \sim N(60^\circ, (0.2^\circ)^2)$. The measurement uncertainty is independent of that in the mechanism dimensions. Use the First Order Second Moment (FOSM) method to calculate the probability of failure.



Solution:

The vector of random variables is $\mathbf{X} = (l_1, l_2, \theta)$. The actual position of the slider is given by

$$l_3 = \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\pi - \theta)} = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta}$$

Since the tolerance range is $l_{3a} \pm 0.027$, failure occurs when $l_{3a} > l_{3a} + 0.027$ or $s < l_{3a} - 0.027$. Let $Y_1 = g_1(\mathbf{X}) = (l_{3a} + 0.027) - l_3$, and $Y_2 = g_2(\mathbf{X}) = l_3 - (l_{3a} - 0.027)$. Then, the probability of failure of the mechanism is given by

$$p_f = \Pr(Y_1 < 0) + \Pr(Y_2 < 0)$$

Since the ransom variables are independent and follow normal distributions, according to the FOSM method, the mean and standard deviation of Y_1 are calculated by

$$\mu_{Y_1} = g_1(\boldsymbol{\mu}_{\mathbf{X}}) = (l_{3a} + 0.027) - \sqrt{\mu_{l_1}{}^2 + \mu_{l_2}{}^2 + 2\mu_{l_1}\mu_{l_2}\cos(\mu_\theta)} = 0.0242\,\text{m}$$

$$\frac{\partial g_1}{\partial l_1} = -\left(\frac{1}{2}\frac{1}{\sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos\theta}}\left(2l_1\right)(2l_2\cos\theta)\right)\Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = -0.9042$$

$$\frac{\partial g_1}{\partial l_2} = -\left(\frac{1}{2}\frac{1}{\sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos\theta}}\left(2l_2\right)(2l_1\cos\theta)\right)\Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = -0.822$$

$$\frac{\partial g_1}{\partial \theta} = -\left(\frac{1}{2}\frac{1}{\sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos\theta}}\left(-2l_1l_2\sin\theta\right)\right)\Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = 1.7085$$

$$\begin{aligned}\sigma_{Y_1} &= \sqrt{\left(\frac{\partial g_1}{\partial l_1}\right)^2\sigma_{l_1}^2 + \left(\frac{\partial g_1}{\partial l_2}\right)^2\sigma_{l_2}^2 + \left(\frac{\partial g_1}{\partial \theta}\right)^2\sigma_\theta^2} \\ &= \sqrt{\left(-0.9042\right)^2(0.002)^2 + \left(-0.822\right)^2\left(0.001\right)^2 + \left(1.7085\right)^2\left(0.0035\right)^2} = 0.0063\,\text{m}\end{aligned}$$

Thus, we have

$$\Pr(Y_1 < 0) = \Phi\left(-\frac{\mu_{Y_1}}{\sigma_{Y_1}}\right) = \Phi(-3.8559) = 5.766\times10^{-5}$$

Similarly, the mean and standard deviation of Y_2 are calculated by

$$\mu_{Y_2} = g_2(\boldsymbol{\mu}_{\mathbf{X}}) = \sqrt{\mu_{l_1}{}^2 + \mu_{l_2}{}^2 + 2\mu_{l_1}\mu_{l_2}\cos(\mu_\theta)} - (l_{3a} - 0.027) = 0.0298\,\text{m}$$

$$\frac{\partial g_2}{\partial l_1} = \left(\frac{1}{2}\frac{1}{\sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos\theta}}\left(2l_1\right)(2l_2\cos\theta)\right)\Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = 0.9042$$

$$\frac{\partial g_2}{\partial l_2} = \left(\frac{1}{2}\frac{1}{\sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos\theta}}\left(2l_2\right)(2l_1\cos\theta)\right)\Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = 0.822$$

$$\frac{\partial g_2}{\partial \theta} = \left(\frac{1}{2} \frac{1}{\sqrt{l_1{}^2 + l_2{}^2 + 2l_1 l_2 \cos\theta}} \left(-2l_1 l_2 \sin\theta \right) \right) \Bigg|_{(l_1=\mu_{l_1},\ l_2=\mu_{l_2},\theta=\mu_\theta)} = -1.7085$$

$$\sigma_{Y_2} = \sqrt{\left(\frac{\partial g_1}{\partial l_1} \right)^2 \sigma_{l_1}^2 + \left(\frac{\partial g_1}{\partial l_2} \right)^2 \sigma_{l_2}^2 + \left(\frac{\partial g_1}{\partial \theta} \right)^2 \sigma_\theta^2}$$

$$= \sqrt{\left(-0.9042\right)^2 (0.002)^2 + \left(-0.822\right)^2 \left(0.001\right)^2 + \left(1.7085\right)^2 \left(0.0035\right)^2} = 0.0063\,\text{m}$$

We have

$$\Pr(Y_2 < 0) = \Phi\left(-\frac{\mu_{Y_2}}{\sigma_{Y_2}} \right) = \Phi(-4.7348) = 1.0962 \times 10^{-6}$$

The probability of failure of this mechanism is then given by

$$p_f = \Pr(Y_1 < 0) + \Pr(Y_2 < 0) = 5.766 \times 10^{-5} + 1.0962 \times 10^{-6} = 5.8756 \times 10^{-5}$$ **Ans.**