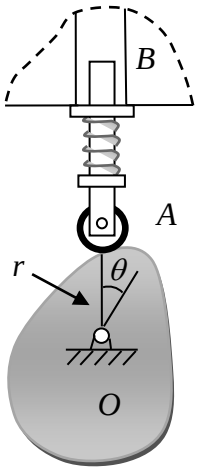
Example 4

A cam system is shown in the figure, in which the cam rotates with a clockwise angular velocity of $\omega \sim N(5, 0.1^2)$ rad/s at the instant $\theta = \frac{\pi}{4}$. The surface of the cam has a shape of a limacon defined by $r = (180 + 90\cos\theta)$ mm. Determine the distribution of the velocity of the follower rod *AB*. If the allowable velocity of *AB* is $v_a = 340$ mm/s, find the probability of failure of the system.



Solution:

The velocity of rod *AB* is calculated by

$$v_{AB} = \dot{r} = -90\sin\theta(\omega) = 63.64\ \omega \downarrow$$

Since $\omega \sim N(5, 0.1^2)$, v_{AB} also follows a normal distribution with

$$\mu_{v_{AB}} = 63.64\ \mu_\omega = 318.2 \text{ mm/s}$$

$$\sigma_{v_{AB}} = 63.64\ \sigma_\omega = 6.36 \text{ mm/s}$$

Let $Y = v_a - v_{AB}$, the mean and standard deviation of Y are then given by

$$\mu_Y = v_a - \mu_{v_{AB}} = 340 - 318.2 = 21.8 \text{ mm/s}$$

$$\sigma_Y = \sigma_{v_{AB}} = 6.36 \text{ mm/s}$$

Thus, the probability of failure of the cam system is given by

$$p_f = \Pr(Y < 0) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi(-3.4258) = 3.0644 \times 10^{-4}$$ **Ans.**