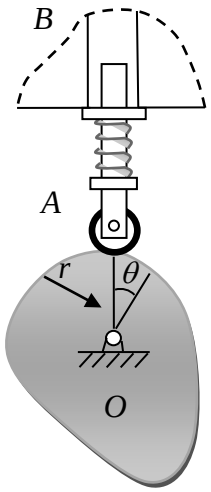
Example 5

The surface of a cam is expressed by a logarithmic spiral formula $r = 35e^{0.05\theta}$ mm, where θ is in radians. The cam rotates at an angular velocity of $\omega \sim N(3, 0.2^2)$ rad/s . Determine the distribution of the velocity of the follower rod *AB* at the instant $\theta = \frac{\pi}{6}$. If the allowable velocity of *AB* is $v_a = 140$ mm/s, find the probability of failure of the system.



Solution:

The velocity of rod *AB* is calculated by

$$v_{AB} = \sqrt{v_r^2 + v_\theta^2}$$

in which

$$v_r = \dot{r} = 35(0.05)e^{0.05\theta}(\dot{\theta})\Big|_{\theta=\frac{\pi}{6}} = 1.75e^{0.0262}(\omega) = 1.80\omega$$

$$v_\theta = r\dot{\theta} = 35e^{0.05\theta}(\dot{\theta})\Big|_{\theta=\frac{\pi}{6}} = 35e^{0.0262}(\omega) = 35.93\omega$$

Thus, v_{AB} could be rewritten as

$$v_{AB} = \sqrt{v_r^2 + v_\theta^2} = \sqrt{1.8^2 + 35.93^2}\ \omega = 35.97\ \omega$$

Since $\omega \sim N(3, 0.2^2)$, v_{AB} also follows a normal distribution with

$$\mu_{v_{AB}} = 35.97\ \mu_\omega = 107.92 \text{ mm/s}$$

$$\sigma_{v_{AB}} = 35.97\ \sigma_\omega = 7.19 \text{ mm/s}$$

Let $Y = v_a - v_{AB}$, the mean and standard deviation of Y are then given by

$$\mu_Y = v_a - \mu_{v_{AB}} = 140 - 107.98 = 32.08 \text{ mm/s}$$

$$\sigma_Y = \sigma_{v_{AB}} = 7.19 \text{ mm/s}$$

Thus, the probability of failure of the cam system is given by

$$p_f = \Pr(Y < 0) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi(-4.4589) = 4.1193 \times 10^{-6}$$ **Ans.**