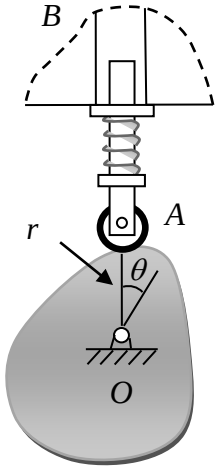
Example 6

A cam system is shown in the figure, in which the cam rotates with a clockwise angular velocity of $\omega = 3$ rad/s at the instant $\theta = \frac{\pi}{6}$. The angular acceleration of the cam follows a normal distribution of $\alpha \sim N(3, 0.2^2)$ rad/s^2. The surface of the cam has a shape of a limacon defined by $r = (200 + 120\cos\theta)$ mm. Determine the distribution of a_r, which is the acceleration of the follower rod *AB*. If the allowable acceleration is $a = -3270$ mm/s^2, the system fails when the magnitude of a_r is larger than that of a. Find the probability of failure of the system.



Solution:

At the instant $\theta = \frac{\pi}{6}$, we have

$$r = (200 + 120\cos\theta) = 303.92 \text{ mm}$$

The time derivatives are calculated by

$$\dot{r} = -120\sin\theta(\omega) = -180 \text{ rad/s}$$

$$\ddot{r} = -120\left[\cos\theta(\omega) + \sin\theta(\alpha)\right] = -60\alpha - 311.77$$

The acceleration of rod *AB* is given by

$$a_r = \ddot{r} - r\omega^2 = -60\alpha - 311.77 - 303.92(3)^2 = -60\alpha - 3047.08$$

Since $\alpha \sim N(3, 0.2^2)$ rad/s^2, a_r also follows a normal distribution with

$$\mu_{a_r} = -60\mu_\alpha - 3047.05 = -3227.08 \text{ mm/s}^2$$

$$\sigma_{a_r} = -60\sigma_\alpha = -12 \text{ mm/s}^2$$

Let $Y = |a| - |a_r|$, the mean and standard deviation of Y are then given by

$$\mu_Y = |a| - |\mu_{a_r}| = 3270 - 3227.08 = 42.92 \text{ mm/s}$$

$$\sigma_Y = \sigma_{a_r} = -12 \text{ mm/s}^2$$

Thus, the probability of failure of the cam system is given by

$$p_f = \Pr(Y < 0) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi(-3.577) = 1.7381\times10^{-4}$$ **Ans.**