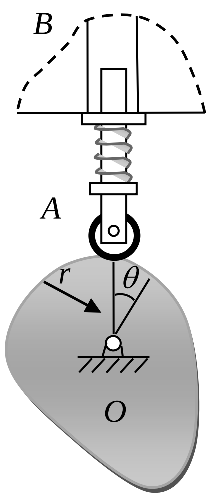
Example 7

The surface of a cam is expressed by a logarithmic spiral formula $r = ke^{0.05\theta}$ mm, where θ is in radians. Due to the uncertainty in the manufacturing process of the cam, the coefficient k follows a normal distribution of $k \sim N(30, 0.1^2)$. The cam rotates at an constant angular velocity of $\omega = 5$ rad/s. Determine the distribution of the velocity and the acceleration of the point on the cam that contacts the follower rod *AB* at the instant $\theta = \frac{\pi}{6}$. If the allowable acceleration of *AB* is $a = 800$ mm/s^2, find the probability of failure of the system.



Solution:

The velocity of the contact point on the cam is calculated by

$$v_c = \sqrt{v_r^2 + v_\theta^2}$$

in which

$$v_r = \dot{r} = k\,(0.05)e^{0.05\theta}(\omega)\Big|_{\theta=\frac{\pi}{6},\omega=5} = 0.27k$$

$$v_\theta = r\dot{\theta} = ke^{0.05\theta}(\omega)\Big|_{\theta=\frac{\pi}{6},\omega=5} = 5.13k$$

Thus, v_c could be rewritten as

$$v_c = \sqrt{v_r^2 + v_\theta^2} = 5.14k$$

Since $k \sim N(30, 0.1^2)$, v_c follows a normal distribution with

$$\mu_{v_c} = 5.14\ \mu_k = 154.2\ \text{mm/s}$$

$$\sigma_{v_c} = 5.14\ \sigma_k = 0.51\ \text{mm/s}$$ **Ans.**

The acceleration of the contact point on the cam is calculated by

$$a_c = \sqrt{a_r^2 + a_\theta^2}$$

in which

$$a_r = \ddot{r} - r(\omega)^2 = k(0.05)\left[0.05e^{0.05\theta}(\omega)^2 + e^{0.05\theta}(\dot{\omega})\right] - ke^{0.05\theta}(\omega)^2\Bigg|_{\theta=\frac{\pi}{6},\omega=5,\dot{\omega}=0} = -30.73k$$

$$a_\theta = r\dot{\omega} + 2\dot{r}\omega = 2k(0.05)e^{0.05\theta}(\omega)^2\Big|_{\theta=\frac{\pi}{6},\omega=5,\dot{\omega}=0} = 2.57k$$

Thus, a_c could be represented by

$$a_c = \sqrt{a_r^2 + a_\theta^2} = 30.83k$$

which also follows a normal distribution with

$$\mu_{a_c} = 30.83\ \mu_k = 925.01\ \text{mm/s}^2$$

$$\sigma_{a_c} = 30.83\ \sigma_{a_c} = 3.08\ \text{mm/s}^2$$ **Ans.**

Let $Y = a - a_c$, the mean and standard deviation of Y are then given by

$$\mu_Y = a - \mu_{a_c} = 940 - 925.01 = 14.99\ \text{mm/s}^2$$

$$\sigma_Y = \sigma_{a_c} = 3.08\ \text{mm/s}^2$$

Thus, the probability of failure of the cam system is given by

$$p_f = \Pr(Y < 0) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi(-4.8606) = 5.8514 \times 10^{-7}$$ **Ans.**