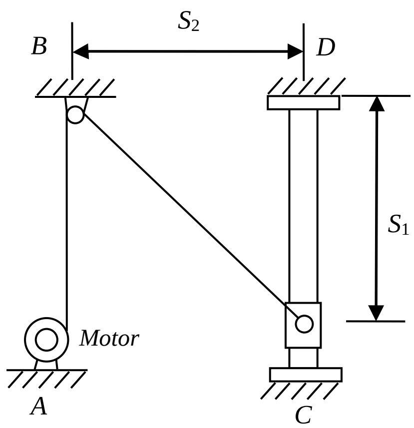
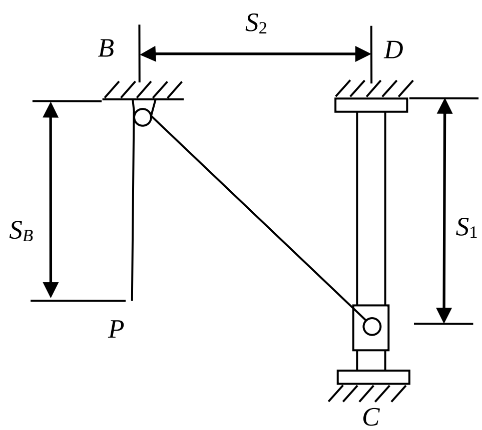
## Example 8

The motion of the collar on beam *CD* is controlled by a motor at *A*. The distance between the collar and point *D* is $S_1$, which follows a normal distribution of $S_1 \sim N(1,\ (5\times10^{-2})^2)$ m due to the uncertainty in measurement. The collar moves upward at a velocity of $v_1 = 0.37$ m/s. The length of $S_2$ also follows a normal distribution of $S_2 \sim N(1.2,\ (5\times10^{-2})^2)$ m. At this instant the allowable velocity of a point on the cable is $v_a = 0.28$ m/s, use FOSM method to find the probability of failure. Assume that $S_1$ and $S_2$ are independent.




**Solution:**

Assume that a point *P* lies in the *AB* part of the cable and the distance between this point and point *B* is $S_B$, as shown in the figure below. Let $l$ denote the distance from the point to the collar along the cable.



According to the figure, we have

$$\sqrt{S_1^2 + S_2^2} + S_B = l$$

Taking the time derivative, we obtain the velocity of the point on the cable.

$$v = \dot{S}_B = -\frac{\dot{S}_1 S_1}{\sqrt{S_1^2 + S_2^2}} = -\frac{v_1 S_1}{\sqrt{S_1^2 + S_2^2}}$$

Let $Y = g(\mathbf{X}) = v_a - v$, and then the probability of failure is given by

$$p_f = \Pr(Y < 0)$$

Since the random variables are independent and follow normal distributions, the mean and standard deviation of $Y$ are calculated by

$$\mu_Y = v_a - \frac{v_1 \mu_{S_1}}{\sqrt{\mu_{S_1}^2 + \mu_{S_2}^2}} = 0.043\,\text{m}$$

$$\frac{\partial Y}{\partial S_1} = -\frac{v_1}{(S_1^2 + S_2^2)^{1/2}} + \frac{v_1 S_1 2 S_1}{(S_1^2 + S_2^2)^{3/2}}\Bigg|_{(S_1 = \mu_{S_1},\, S_2 = \mu_{S_2})} = -0.14\ \text{m}$$

$$\frac{\partial Y}{\partial S_2} = \frac{v_1 S_1 2 S_2}{(S_1^2 + S_2^2)^{3/2}}\Bigg|_{(S_1 = \mu_{S_1},\, S_2 = \mu_{S_2})} = 0.117\ \text{m}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial Y}{\partial S_1}\right)^2 \sigma_{S_1}^2 + \left(\frac{\partial Y}{\partial S_2}\right)^2 \sigma_{S_2}^2}$$
$$= \sqrt{(-0.14)^2 (0.05)^2 + (0.117)^2 (0.05)^2} = 0.009\,\text{m}$$

Thus, we have

$$\Pr(Y < 0) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi(-4.741) = 1.065 \times 10^{-6}$$ **Ans.**