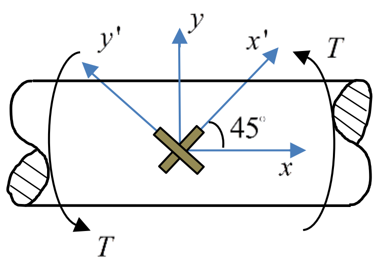
1-2. The radius of a shaft is 14 mm. Two strain gauges are attached to the surface of the shaft as shown in the figure. The strains of $\varepsilon_{x'}$ and $\varepsilon_{y'}$ are measured repeatedly. From the measured results, the distribution of $\varepsilon_{x'}$ is found to be $\varepsilon_{x'} \sim N\left(-50\times10^{-6},(4\times10^{-6})^2\right)$, what is the estimated torque in the form of $\mu_T \pm 2\sigma_T$. Assume that $E = 200\,\text{GPa}$, $v = 0.3$.



Solution:

Pure shear $\varepsilon_x = \varepsilon_y = 0$

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

Use $\theta = 45^\circ$ in the above equation, we have

$$\varepsilon_{x'} = 0 + 0 + \gamma_{xy} \sin 45^\circ \cos 45^\circ$$

Thus

$$\gamma_{xy} = 2\varepsilon_{x'}$$

$$G = \frac{E}{2(1+v)} = \frac{200(10^9)}{2(1+0.3)} = 76.92\times10^9$$

$$\tau = G\gamma_{xy} = 76.92\times10^9\gamma_{xy} = \left(153.84\times10^9\right)\varepsilon_{x'}$$

Then the torque can be given by

$$T = \frac{\tau J}{c} = \frac{\left(153.84\times10^9\right)\left(\dfrac{\pi}{2}(0.014)^4\right)}{0.014} = \left(6.6312\times10^5\right)\varepsilon_{x'}$$

Thus, T also follows a normal distribution with

$$\mu_T = \left(6.6312 \times 10^5\right)\mu_{\varepsilon_{x'}} = \left(6.6312 \times 10^5\right)\left(50 \times 10^{-6}\right) = 33.1559 \text{ N} \cdot \text{m}$$

$$\sigma_T = \left(4.17 \times 10^{-4}\right)\sigma_{\varepsilon_{x'}} = \left(6.6312 \times 10^5\right)\left(4 \times 10^{-6}\right) = 2.6525 \text{ N} \cdot \text{m}$$

The estimated torque T is $T = 33.1559 \pm 2(2.6525) \text{ N} \cdot \text{m}$. **Ans.**