Robust Design and Reliability-Based Design

ME 4761 Engineering Design
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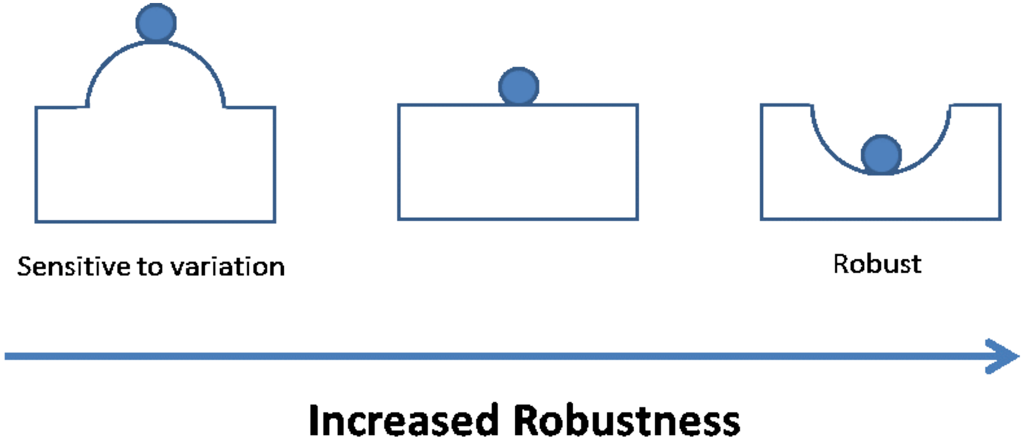
Founded 1870 | Rolla, Missouri

Outline

- Definition of robustness
- Introductory examples
- Statistics
- How to achieve robustness
- Examples
- Related methodology: reliability-based design
- Conclusions

Robust Design

- If a design can work properly even when subjected to variation, it is robust.
- Variation (uncertainty) may be introduced by
  - manufacturing processes
  - environment
  - parts from outside suppliers
  - end user

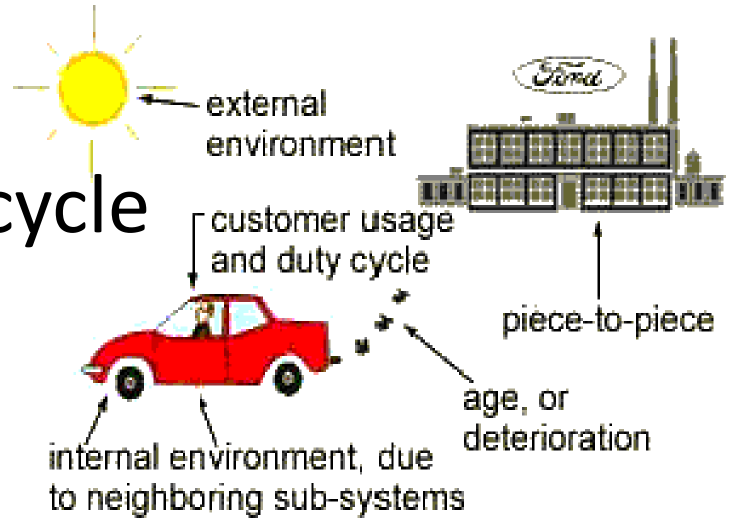


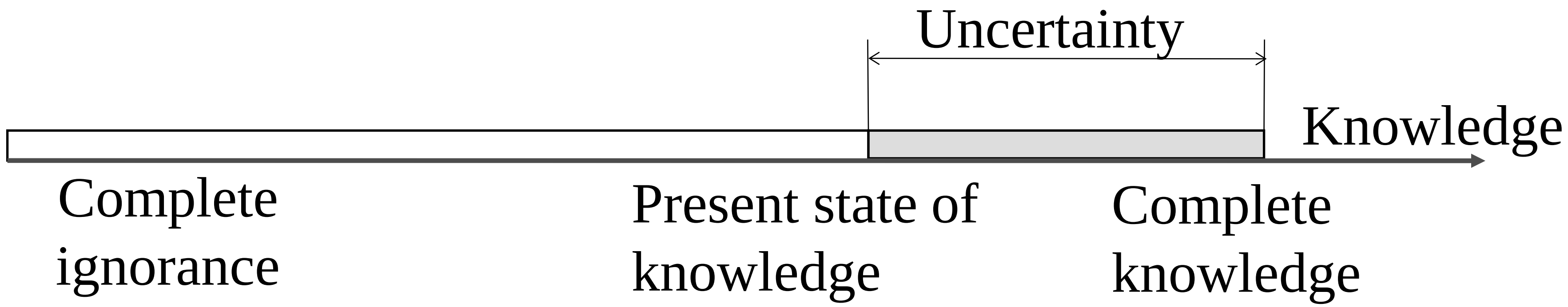
Robustness

- The robustness of a product is the ability that its performances are not affected by the uncertain inputs or environment conditions (noises).
- Robustness is insensitivity to uncertainty.
- A robust product can work under large uncertainties.

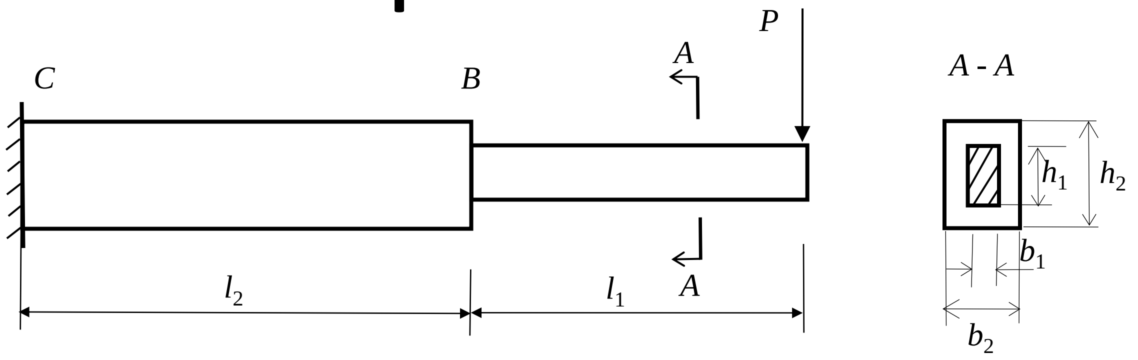
Variation (Uncertainty)

- Piece-to-piece variation
- Customer usage and duty cycle
- Human errors
- Model inaccuracy





An Example: Cantilever Beam



The material strength S

The maximum stress $\sigma_{\max} = \dfrac{6P(l_1 + l_2)}{b_2 h_2^2}$

Factor of safety $F_s = \dfrac{S}{\sigma_{\max}} > 1$

Reality: everything is uncertain
Load: P=(2001.4, 1531.3, 2534.6, ….) kN
Yield strength: Sy=(120.5, 101.3, 131.2, 170.9,…) MPa
Dimension: h_1= 100±0.01 mm
Dimension: b_1= 50±0.01 mm, ….

Principle of Robust Design

- Minimize the effect of uncertainty (variation) without eliminating the cause of uncertainty.
- How?
- By changing design variables to make the performance not sensitive to uncertainty.

Example: Tile Manufacturing

- Output: Tile dimensions
- Problem: Large variability in dimensions
- Uncertainty source: Huge variation in temperature
- Possible solutions
  - Screening
  - Redesign the kiln
  - Too expensive
- Robust design solution
  - Do not control the temperature
  - Change design variables: increasing the lime content of the clay from 1% to 5%
  - Inexpensive

Phadke: Quality Engineering Using Robust Design, 1989

Benefits of Robust Design

- Product performance insensitive to material variation –> use of low grade material and components
- Product performance insensitive to manufacturing variation -> reduced labor and manufacturing cost
- Product performance insensitive to the variation in operating environment -> higher reliability and lower operation cost

How Do We Quantify Uncertainty?

- Support we have 100 measurements for $X = L_1$
- (99.99, 100.08,…,100.05) mm

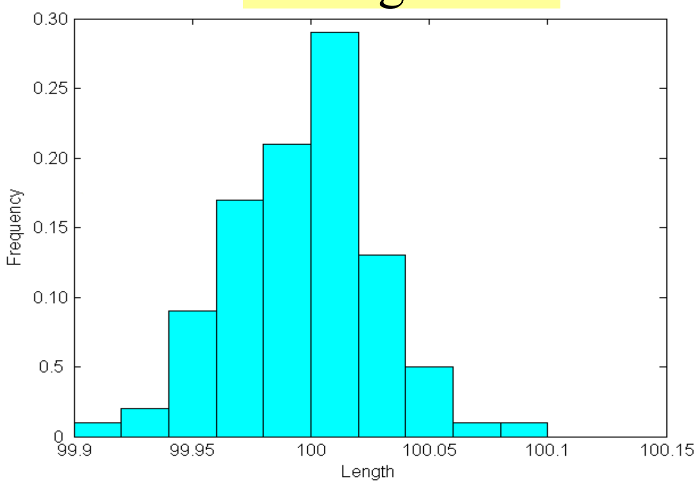
Average

$$\text{Mean } \overline{X} = \frac{1}{100}\sum_{i=1}^{100} x_i = 99.96$$

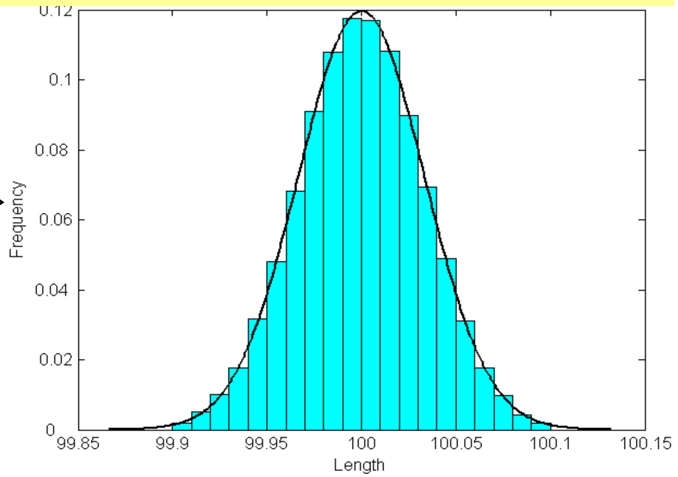
Dispersion

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{100-1}\sum_{i=1}^{n}(x_i - \overline{X})^2}$$

Histogram



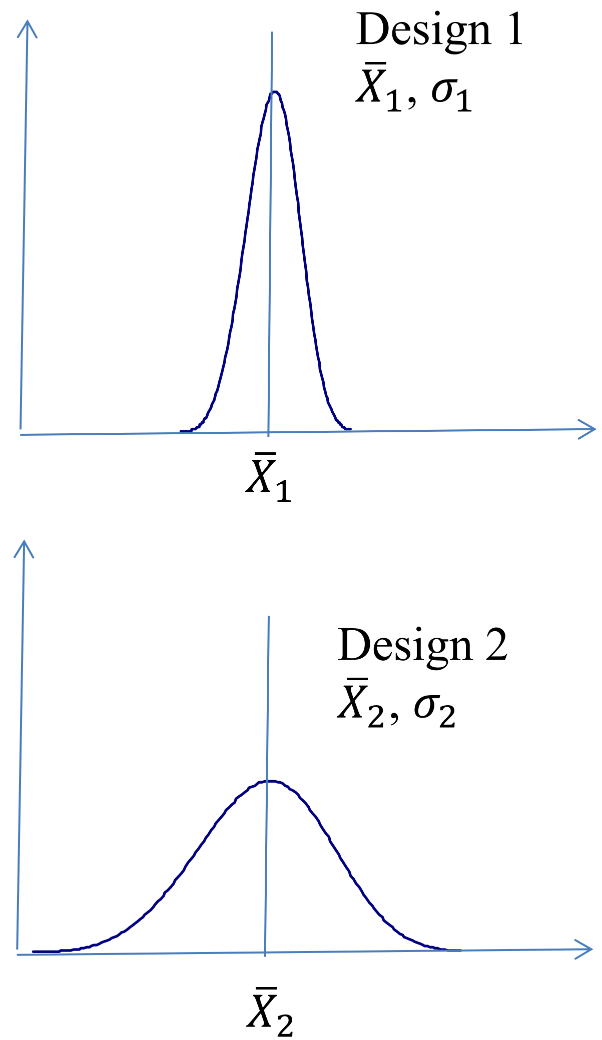
Probability density (distribution)



Probability Distribution

- Mean – average
- Standard deviation (std) – dispersion around the average or amount of variation

- Two designs with two performance variables
- $\bar{X}_1 = \bar{X}_2, \sigma_2 > \sigma_1$
- Designs 1 and 2 have the same average performance
- The variation of Design 1 is smaller
- Design 1 is more robust



TV Example

- In 1970s, Americans showed a preference for television sets made by Sony-Japan over those made by Sony-USA.
- The color density was a major performance.
  - Target ± tolerance = m ± 5.
- Sony-Japan sets: 0.3% defective sets (outside the tolerance limits)
- Sony-USA sets: virtually NO sets outside the tolerance limits.
- Why?

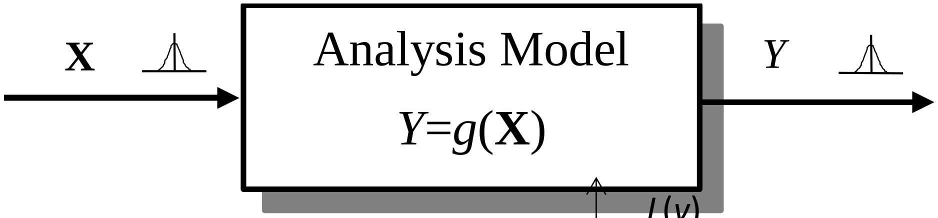
Phadke: Quality Engineering Using Robust Design, 1989

TV Example

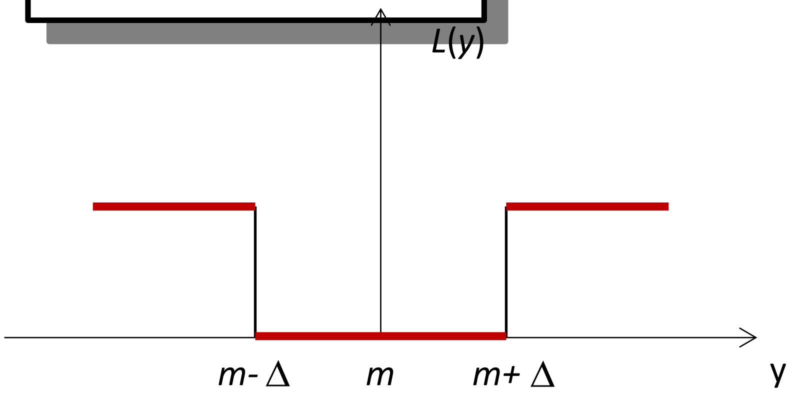
- Sony-USA: Uniform distribution, $\sigma = 2.89$
- Sony-Japan: Normal distribution, $\sigma = 1.67$, and most of the sets are grade A.

Phadke: Quality Engineering Using Robust Design, 1989

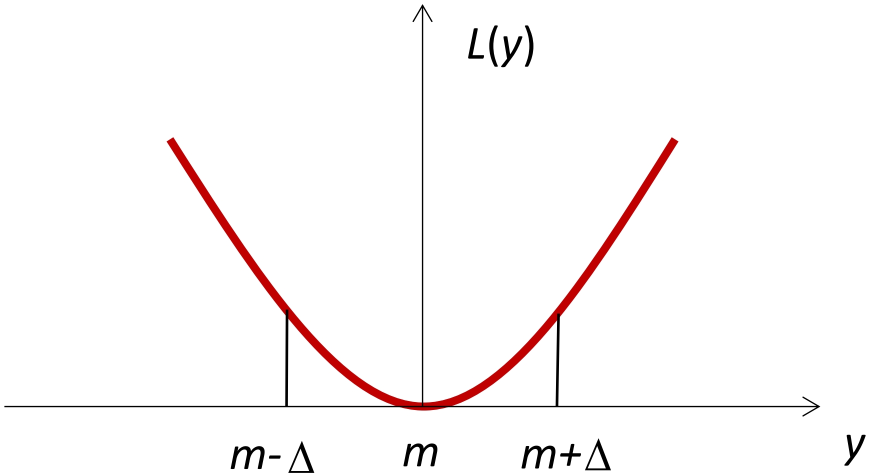
How to Evaluate Robustness?



- Traditional design
  - *m*: target, Δ: tolerance
  - *L*(*y*): quality loss = 0 within the tolerance limits



- Robust design
  - Taguchi's quality loss ($) $L(y) = k(y - m)^2$, *k*: constant
  - quality loss = 0 only at *m*



Expected Quality Loss

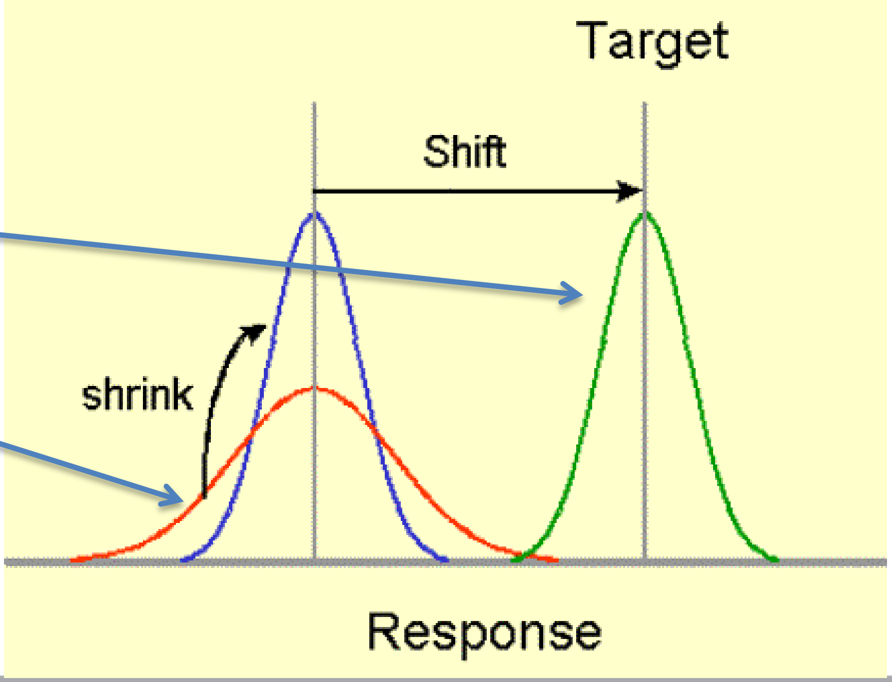
- Expected (average) quality loss

$$E(L) = k[(\bar{Y} - m)^2 + \sigma_Y^2]$$

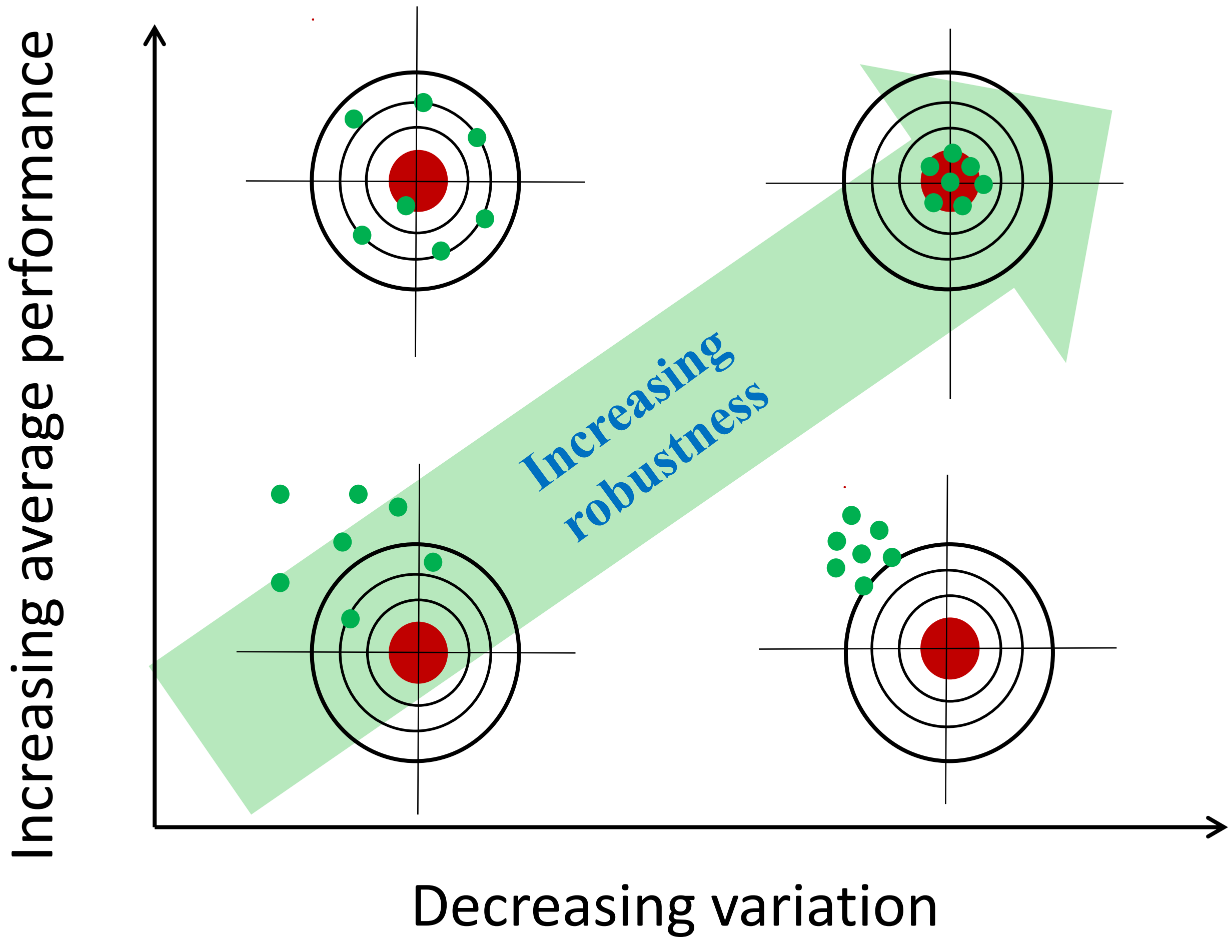
- Minimizing $E(L)$ will bring the average performance to the target and reducing variation σ_Y simultaneously.

Final design

Initial design

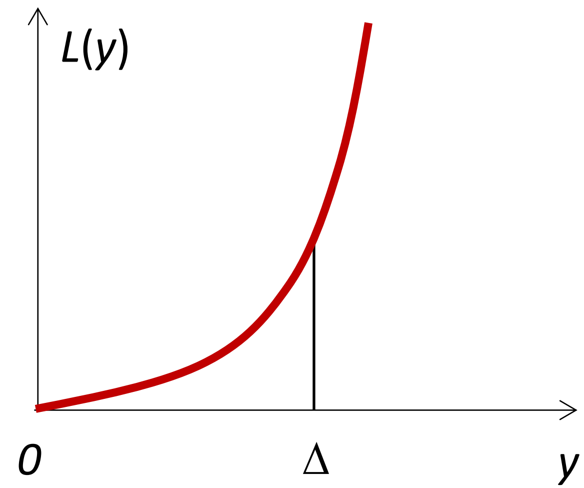


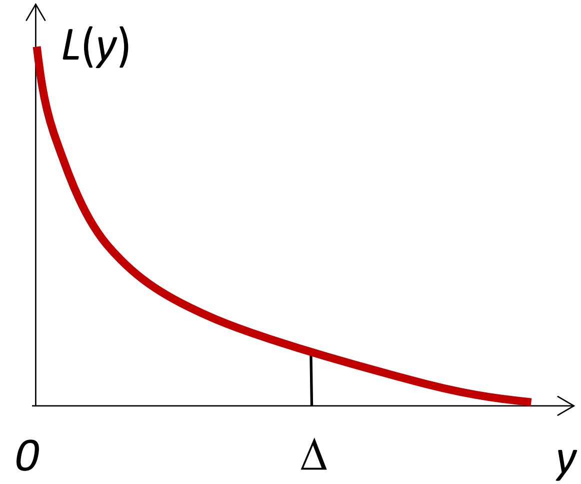
Robust Design



Other Types of Quality Loss

- What we've discussed is the "nominal-the-better" type
- The "smaller-the-better" type
  - cost, stress, energy consumption





- The "larger-the-better" type
  - life, reliability, strength, efficiency

How to Select Design Variables to Achieve Robustness?

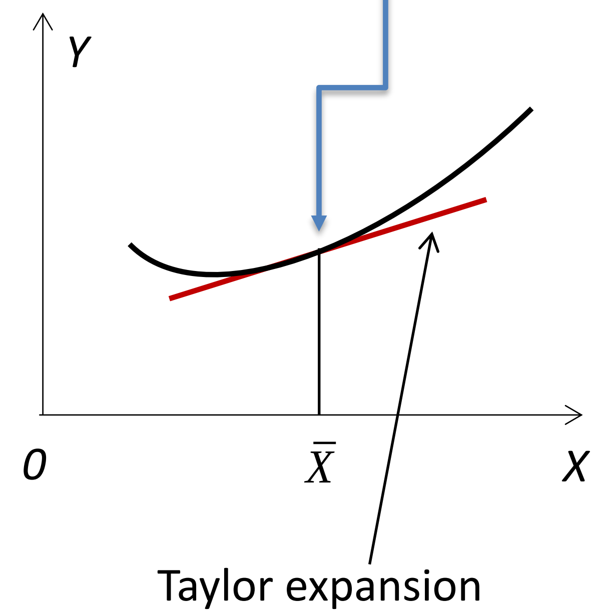
- Performance $Y = g(X_1, X_2, \ldots, X_n)$
- Design variables $\bar{\mathbf{X}} = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$
- $X_i\ (i = 1,2,\cdots,n)$ are independent

Parameter Design

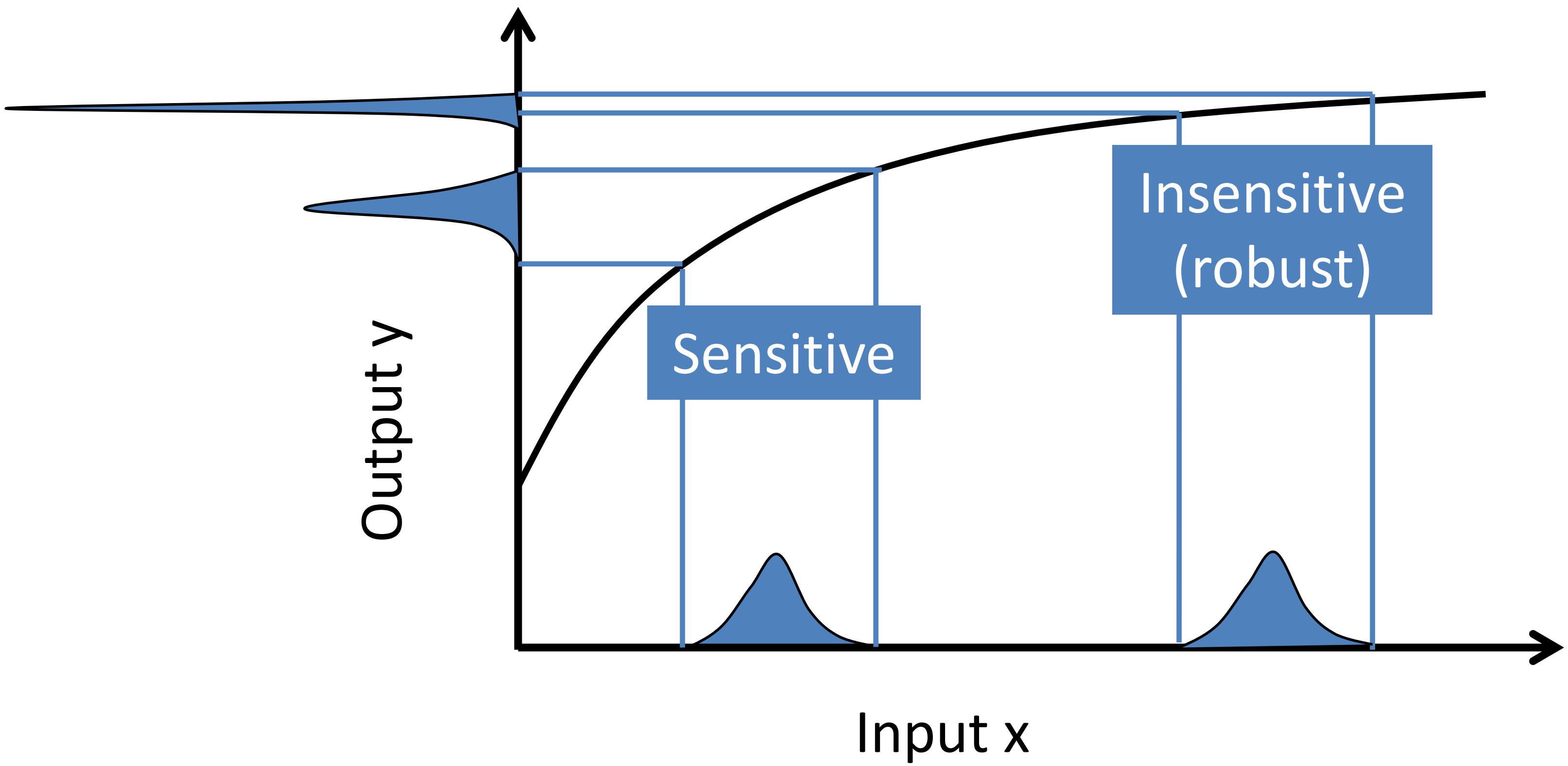
- Average performance $\bar{Y} = f(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$
- Taylor expansion series
- $Y \approx \bar{Y} + c_1(X_1 - \bar{X}_1) + c_2(X_2 - \bar{X}_2) + \cdots + c_n(X_n - \bar{X}_n)$
- $c_i = \frac{\partial g}{\partial X_i}$ at $\bar{\mathbf{X}}$ $(i = 1,2,\cdots,n)$
- Std $\sigma_Y = \sqrt{c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \cdots + c_n^2\sigma_n^2}$
- Change $\bar{\mathbf{X}}$ (not reduce σ_i) to minimize

$$\mathrm{E}(L) = k[(\bar{Y} - m)^2 + \sigma_Y^2]$$

Sensitivity

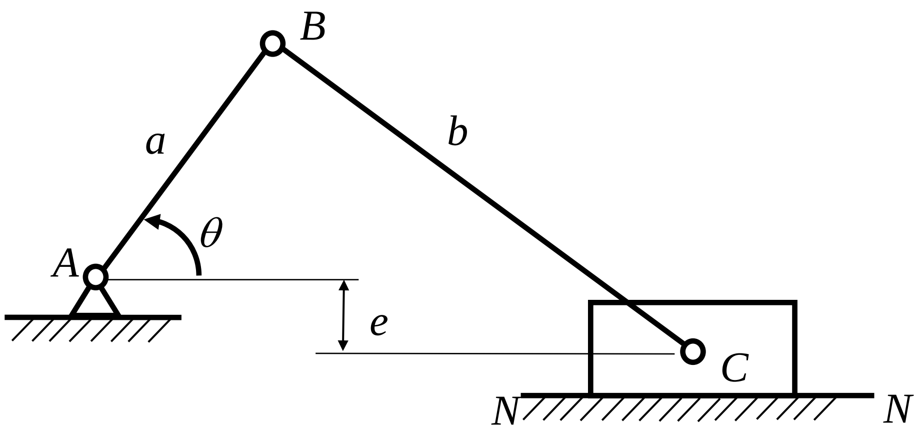


More about Sensitivity



Example: Robust Mechanism Synthesis

- Requirements:
  $s = m_1 = 350$ mm when $\theta = 10°$
  $s = m_2 = 250$ mm when $\theta = 60°$
- Uncertainties in a, b, and e
  $\sigma_a = 2$ mm, $\sigma_b = 2$ mm, $\sigma_e = 3$ mm
- Design variables
  $\bar{a}, \bar{b}, \bar{e}$



Results

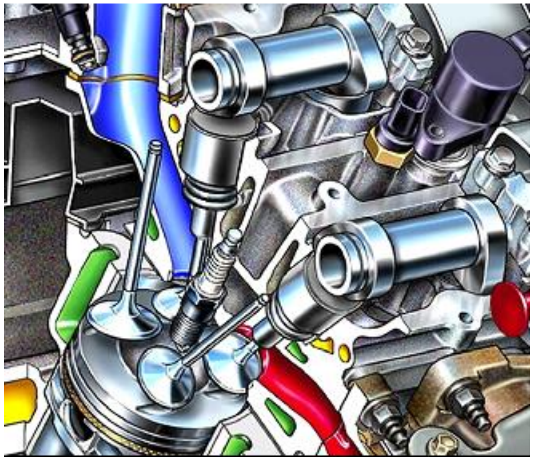
$$s = a\cos\theta + \sqrt{b^2 - (e + a\sin\theta)^2}$$

Transmission angle > 45°

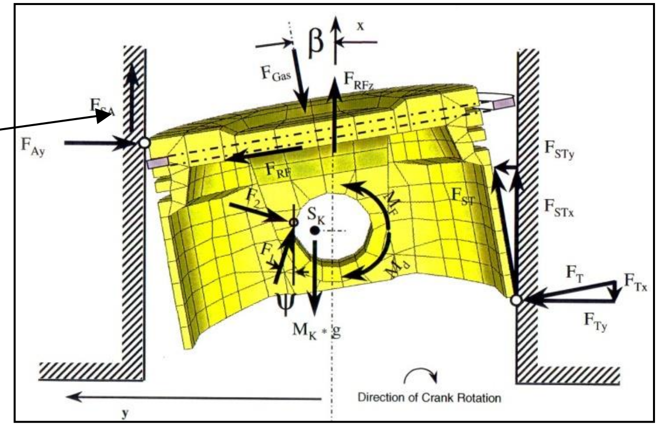
Minimize $\sum_{i=1}^{2} k[(\bar{s}_i - m_i)^2 + \sigma_{si}^2]$

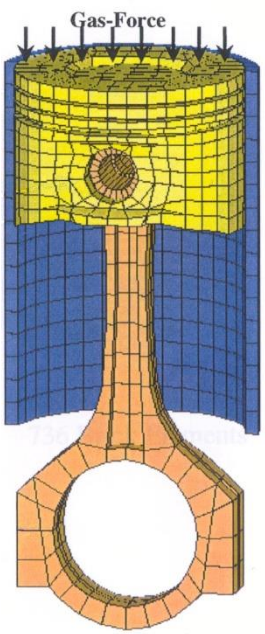
Method	Deterministic Design	Robust Design
$\bar{a}$ (mm)	119.6	136.6
$\bar{b}$ (mm)	241.3	216.8
$\bar{e}$ (mm)	45.0	0.0
$\bar{s}$ $(\theta = 10°)$ (mm)	350	350
$\bar{s}$ $(\theta = 60°)$ (mm)	250	250
σ $(\theta = 10°)$ (mm)	2.9	2.8
σ $(\theta = 60°)$ (mm)	3.5	3.1

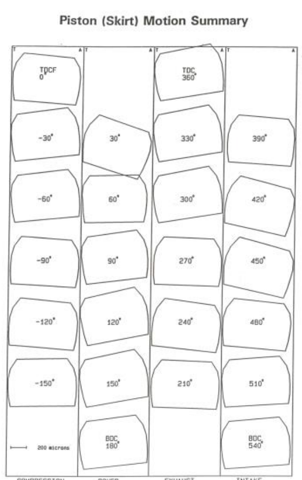
Example 2 - Piston Engine Robust Design



Liner







$$\min_{\boldsymbol{\mu}_{\mathbf{x}}} \quad w_1\mu_f + w_2\sigma_f$$

$$s.t.\ P\{G < 7\} \geq 0.99$$

f – Slap noise

G - Friction

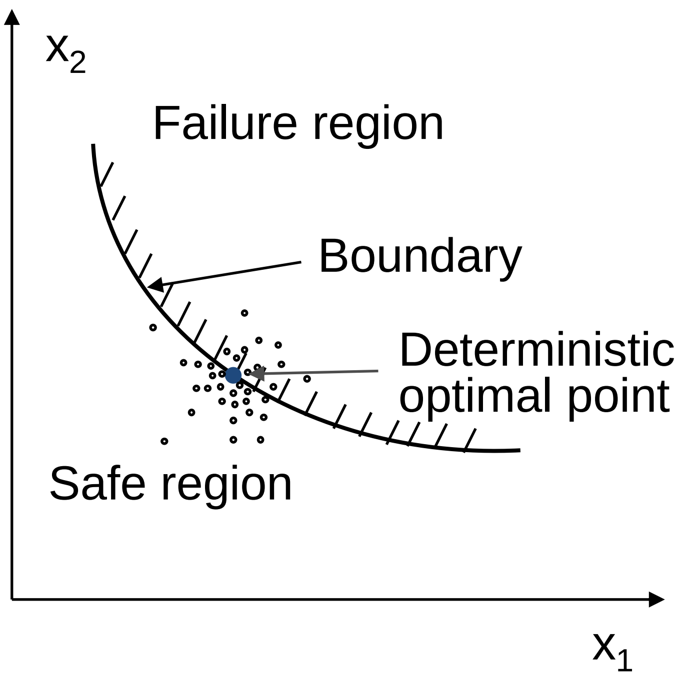
	Baseline	Optimal
Mean of *f*	54.5 dB	54.2 dB
Std of *f*	2.04 dB	0.76 dB
Prob	0.65	0.99

Other Method
Operating Window Methods

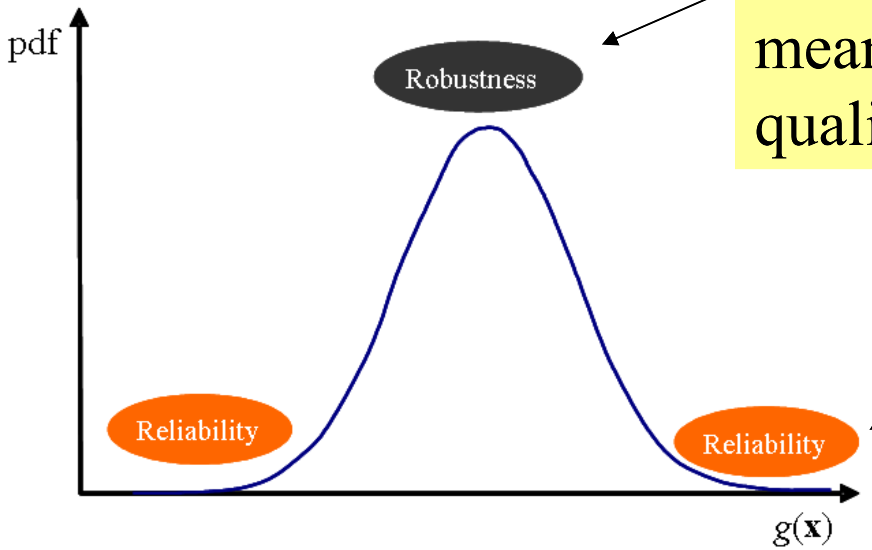
- Developed by Xerox.
- The operating window is the set of conditions under which the system operates without failure.
- A wider window can accommodate larger uncertainties.
- Robustness is achieved by making the operating window larger.

Related Issue: Reliability (R)

- The ability of a product performing its intended function
- $R =$ probability of no failure
- $R = \Pr\{\text{design points falling into safe region}\}$
- $R = \Pr\{\text{strength} > \text{stress}\}$
- $R = \Pr\{\text{factor of safety} > 1\}$



Reliability vs. Robustness



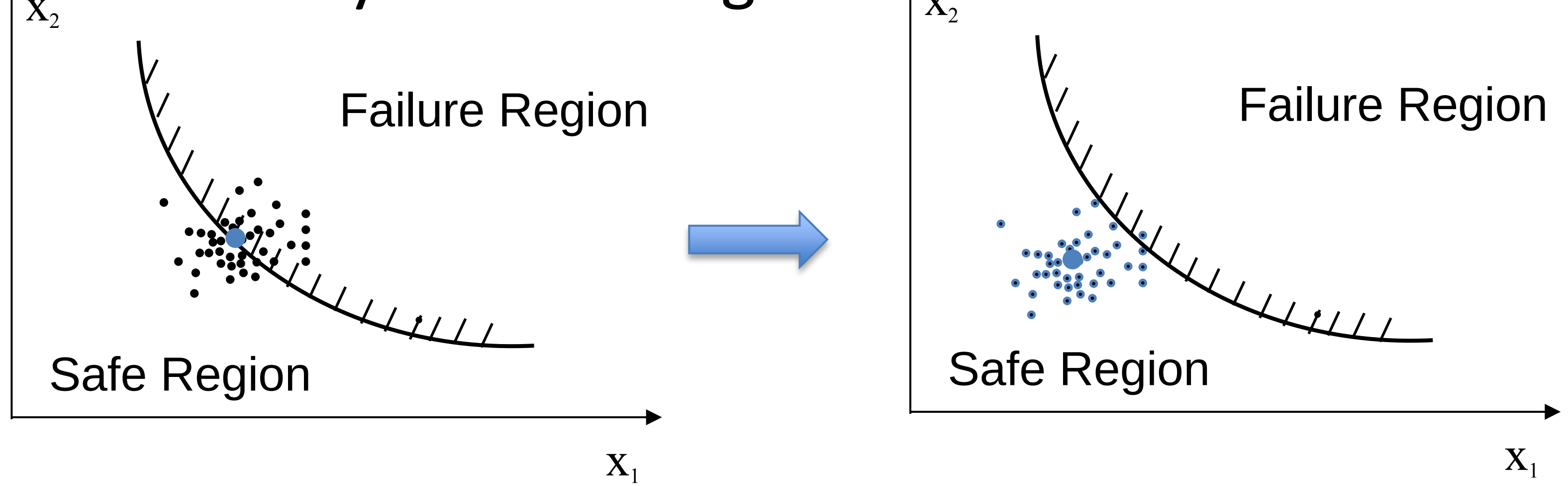
Everyday fluctuations around the mean – deterioration, degradation, quality loss

Extreme events at tails – failure, catastrophe

- Reliability issue
  - Motherboard failure
  - Broken hard disk
- Robustness issue
  - Overheating
  - Noise

How to Design for Reliability?

- Conceptual design
  - Failure mode and effects analysis (FMEA) (IDE 20 & ME 161)
- Parameter design
  - Reliability-based design



Software Tools

- iSIGHT
- MSC – Robust Design
  - **Robust Design for Whirlpool Products**
  - http://www.mscsoftware.com/success/details.cfm?Q=285&sid=282
- Ansys – Monte Carlo Simulation
- ADAMS – Design of Experiments

Conclusions

- Robust design -> insensitivity to uncertainties
  - Insensitive to material variations-> use of low grade materials and components -> low material cost
  - Insensitive to manufacturing variations -> no tightened tolerances -> low manufacturing and labor cost
  - Insensitive to variations in operation environment -> low operation cost
- Robust design -> increased performance, quality, and reliability at reduced cost
- Robustness and reliability can be built into products during early stages of design.

More Information

- Visit the website of Engineering Uncertainty Repository at http://www.mst.edu/~dux/repository.
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