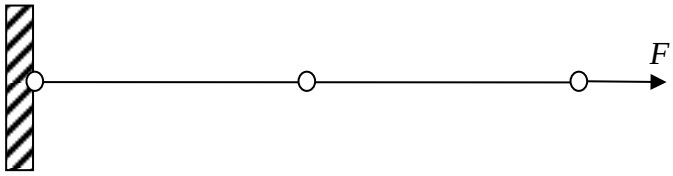
10. A system consists of two cables and is subject to a random force $F \sim \mathrm{N}(\mu_F, \sigma_F^2) = \mathrm{N}(2000, 200^2)$ N as shown in the figure below. The allowable tensions of the two cables are normally distributed with $S_1 \sim \mathrm{N}(\mu_{S_1}, \sigma_{S_1}^2) = \mathrm{N}(3100, 250^2)$ N and $S_2 \sim \mathrm{N}(\mu_{S_2}, \sigma_{S_2}^2) = \mathrm{N}(3000, 150^2)$ N , respectively. All the three random variables are independent. Determine the reliability of the system. (If either cable breaks, the system fails.)



Solution

Let $Y_1 = F - S_1$ and $Y_2 = F - S_2$

$\mu_{Y_1} = \mu_F - \mu_{S_1} = 2000 - 3100 = -1100$

$\mu_{Y_2} = \mu_F - \mu_{S_2} = 2000 - 3000 = -1000$

$\sigma_{Y_1}^2 = \sigma_F^2 + \sigma_{S_1}^2 = 200^2 + 250^2 = 102500$

$\sigma_{Y_2}^2 = \sigma_F^2 + \sigma_{S_2}^2 = 200^2 + 150^2 = 62500$

$\mathrm{Cov}(Y_1, Y_2) = \sigma_F^2 = 200^2 = 40000$

$$\Sigma = \begin{pmatrix} \sigma_{Y_1}^2 & \mathrm{Cov}(Y_1, Y_2) \\ \mathrm{Cov}(Y_1, Y_2) & \sigma_{Y_2}^2 \end{pmatrix} = \begin{pmatrix} 102500 & 40000 \\ 40000 & 62500 \end{pmatrix}$$

(Y_1, Y_2) follows a bivariate normal distribution.

$R_s = F_{Y_1 Y_2}([0,0];[\mu_{Y_1}, \mu_{Y_2}], \Sigma) = 0.999675$