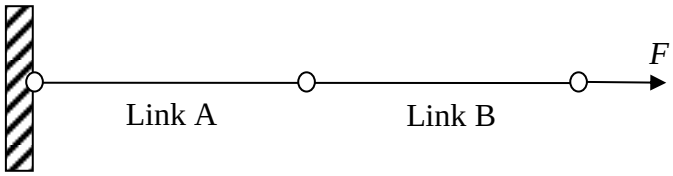
4. A system contains two connected links that are subject to a random force. The probabilities of failure of the links are $P(A)=10^{-4}$ and $P(B)=10^{-4}$. Find the probability of failure of the system.



Solution
Events:
A = Fracture of link A
B = Fracture of link B
C = Fracture of the system

$$P(C)=P(A\cup B)=P(A)+P(B)-P(AB)$$

We don't know $P(AB)$. We could consider two extreme cases.

1) A and B are independent.
$$P_f=P(A)+P(B)-P(A)P(B)$$
$$=10^{-4}+10^{-4}-10^{-4}\left(10^{-4}\right)=0.00019999$$

2) A and B are completely dependent. If one component fails, the other component will fail.
$P(A|B)=1$ or $P(B|A)=1$
$$P_f=P(A)+P(B)-P(A|B)P(B)$$
$$=10^{-4}+10^{-4}-1\left(10^{-4}\right)=0.0001$$

Therefore,
$0.0001\le P_f\le 0.00019999$