8. The time to failure of an electronic component T follows a distribution given below.

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & t \ge 0 \\ 0 & t \ge 0 \end{cases}$$

where $\lambda = 10^{-5}$/hr .

1) What is the probability of failure at $t = 1000$ hr ?
2) What is the 95% percentile value of the time to failure?
3) What is the mean life?
4) What is the standard deviation of the life?

Solution

1) $P(T \le t) = F(t) = \int_{-\infty}^{t} f(t)dt = \int_{-\infty}^{t} \lambda e^{-\lambda t} = 1 - e^{-\lambda t}$

$$P(T \le 1000) = F(1000) = 1 - e^{-1000(10^{-5})} = 0.009950$$

2) $F(t) = 1 - e^{-\lambda t} = 0.95 = 1 - e^{-(10^{-5})t_{95\%}}$

$$t_{95\%} = 299,573 \text{ hrs}$$

3) $\mu = \int_{-\infty}^{\infty} tf(t)dt = \int_{0}^{\infty} t\lambda e^{-\lambda t} dt = \frac{1}{\lambda} = \frac{1}{10^{-5}} = 100,000 \text{ hrs}$

4) $\sigma^2 = \int_{-\infty}^{\infty} (t-\mu)^2 f(t)dt = \int_{0}^{\infty} \left(t - \frac{1}{\lambda}\right)^2 \lambda e^{-\lambda t} = \frac{1}{\lambda^2}$

$$\sigma = \frac{1}{\lambda} = 100,000 \text{ hrs}$$