Chapter One

Introduction

1.1 Introductory Comments

Nowadays engineers are facing new and emerging challenges. Examples of these challenges include the intensive use of computational simulations and virtual prototyping, the application of new technologies into complex systems, the requirements of high quality and reliability, and the reliable decision-making under uncertain design environments. Conventional design methods are inadequate to deal with these challenges. Therefore, nontraditional design methods such as probabilistic engineering design have been increasingly developed and used in industrial applications.

How to deal with uncertainty in engineering is one of the challenges. Uncertainty is ubiquitous in any engineering systems and at any stages of product development. Examples of uncertainty include manufacturing imprecision, usage variation, and imperfect knowledge. To manage various uncertainties, engineers have increasingly applied probabilistic and statistical methods as an integral part in analysis and design activities. However, because graduating engineers typically are not readily equipped with practical knowledge of probabilistic and statistical methods, companies have to provide them with intensive internal training. It is essential for modern engineers to be armed with basic probabilistic and statistical tools for solving complex engineering problems in the face of uncertainty.

This course aims at exploring computational methodologies for engineering design under uncertainty. It is intended for undergraduate seniors and graduate students who are interested in statistical/probabilistic methods and design optimization in engineering analysis and design. It covers reliability analysis, analytical robustness assessment, robust design, reliability-based design, and their engineering applications. Associated outcomes include 1) an ability to model uncertainties for engineering analysis and design, 2) an ability to apply knowledge of statistics and probability to engineering design, 3) an ability to integrate robust design and reliability-based design with CAD/CAE simulations, design optimization, and Design of Experiments (DOE), and 4) an ability to use probabilistic and statistical methods for Design for Six Sigma.

The following specific topics will be covered:

1. Uncertainty modeling – fundamentals of probability and statistics
2. Uncertainty analysis – analysis methodologies that quantify the effect of uncertainty on design performances

    - Reliability analysis
    - Monte Carlo simulation
    - Sensitivity analysis
    - Robustness assessment
3. Probabilistic design – design methodologies that manage and mitigate the effect of uncertainty
    - Introduction to design optimization
    - Reliability-based design
    - Robust design
    - Integrated probabilistic design
4. Case study in Industry (automotive, structural, and mechanical applications)

In this chapter, the basic concepts of engineering design and design process will be discussed first, followed by the introduction to the fundamentals of probabilistic engineering design. In the subsequent chapters, we will primarily discuss three major topics: 1) uncertainty modeling with the application of probability theory, 2) uncertainty analysis that quantifies the effects of uncertainty, and 3) probabilistic engineering design that manages and mitigates the effects of uncertainty.

1.2 Engineering Design and Engineering Design Process

There are many definitions about engineering design. Two of them are listed below.

Engineering design is a process that establishes and defines solutions to new engineering problems, which have been solved before, or new solutions to engineering problems, which have previously been solved, in a different way. The key word is *new problems or new solutions*.

Engineering design is the process of devising a system, component, or process to meet desired needs [1]. It is a decision-making process (often iterative), in which the basic science, mathematics, and engineering sciences are applied to convert resources optimally to meet a stated objective. The key of this definition is *decision-making*.

A general engineering design process involves the following major phases [1-4].

Phase 1: Problem definition – to collect customer needs, clarify design objectives, establish user requirements, and identify constraints.

Phase 2: Conceptual design – to establish functions and design specifications, generate design concepts (alternatives), evaluate design concepts, and select the best design concepts.

Phase 3: Embodiment design – to engineer a solution principle for the selected design concepts by determining the general arrangements and preliminary shapes and materials of all components. Embodiment design is also called preliminary design.

Phase 4: Detail design – to specify all the details of the final design and produce manufacturing drawings and documentation.

The engineering design process is demonstrated in Fig. 1.1. The input of the design process is the customer needs, and the output of the design process is the final design, including manufacturing specifications and all the documentations. The process is dynamic and iterative. Rework is needed among the design phases before a satisfactory final design is reached.

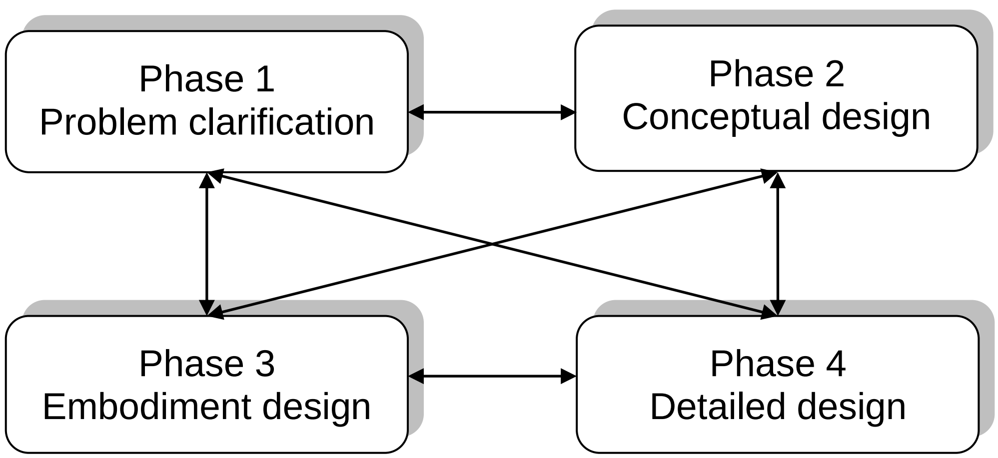

Figure 1.1 General Design Process

1.3 Design and Analysis

Engineering analysis is the study of a design, especially for a product, to understand or model its performance under the conditions of its normal use. It is typically performed on potential designs before they are built or when a product does not meet an expectation. The analysis creates an understanding that allows for improvements in the design and corrections of performance problems. In a design process, analysis is performed to check if an existing design satisfies all the design requirements. Table 1.1 lists the differences between design and analysis.

For example, if a task is to determine a mechanism system that satisfies the functional requirement $y = f(x)$, in which x and y are input and output rotational displacements, respectively. It is a design task because there are multiple solutions to this new problem, and many decisions need to be made, such as the materials, the design options, the geometry, dimensions, etc. Figure 1.2 shows several possible design options to the problem, including a four-bar linkage, a cam-follower mechanism, and a pair of gears.

Table 1.1 Differences between Design and Analysis

Design	Analysis
A decision making process	A problem solving process
Solutions to new problems or new solutions to existing problems	Solutions to existing problems
More than one solution	Only one unique solution

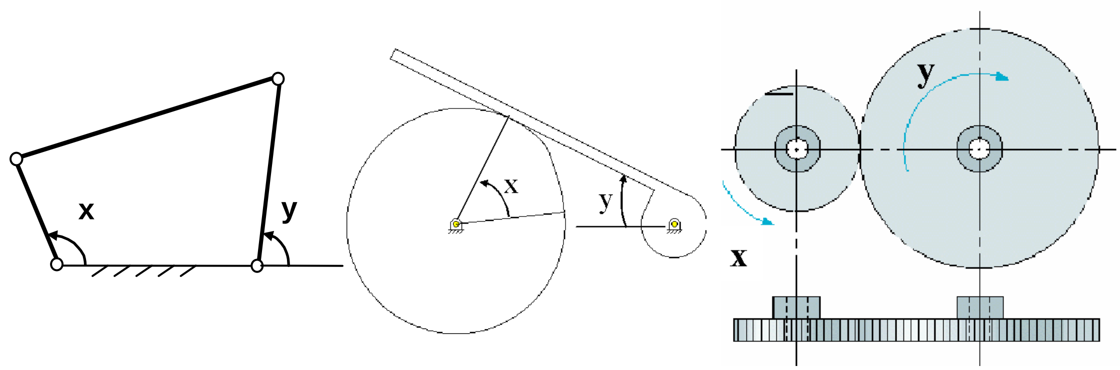

Figure 1.2 Multiple Potential Solutions

Let us look at the reverse problem. A four-bar linkage mechanism has been identified as shown in Figure 1.3. The task is to find the output angle *y* given the input angle *x*. This task is an analysis problem because there is only one unique solution to this existing system, and it is a problem solving process where algebraic equations derived from kinematics are used to find the solution.

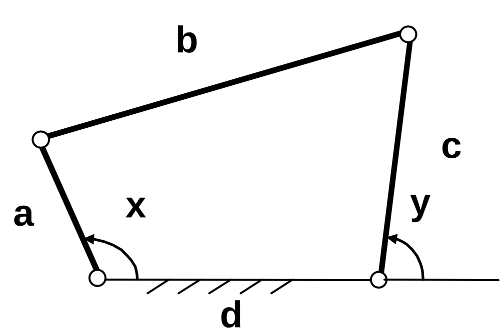

Figure 1.3 Four-Bar Mechanism

We have seen the differences between design and analysis. In a real engineering design process, design and analysis are also tied to each other. A design involves a number of analyses as shown in Fig. 1.4. After having generated a number of design concepts, engineers perform analyses on the design concepts. Then they use the analysis results to make decisions on selecting the best design concepts in terms of design performance. After the concept selection, engineers make more decisions in order to detail and refine the selected design. If the design is not considered satisfactory, they will use the analysis results to improve and update the design by making necessary changes on material selections, configurations, component interfaces, parameters, and so on. The process iterates until a satisfactory design is identified. During this process, numerous decisions are made.

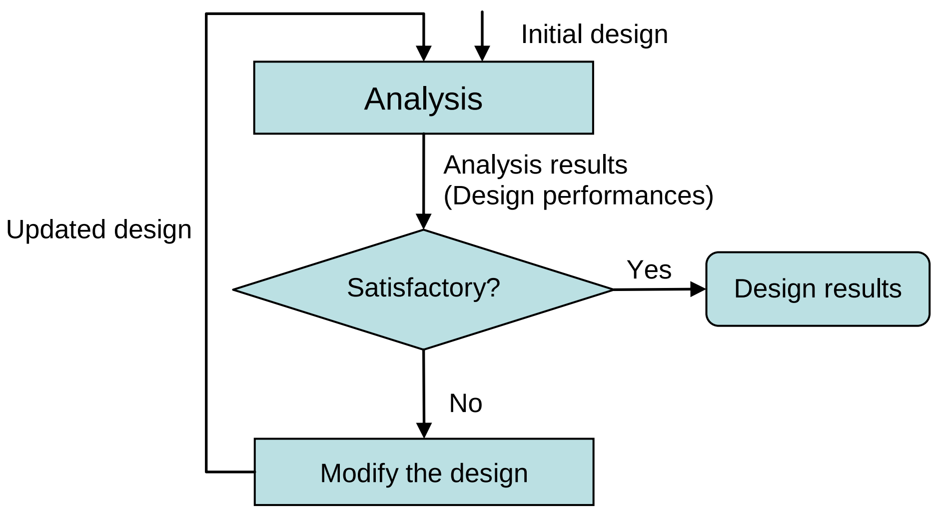

Figure 1.4 Analysis in A Design Process

On the other hand, a single analysis may also contain other design tasks. For example, solving a mathematical equation is an analysis problem. Designing and selecting algorithms to solve the equation is a design problem.

1.4 Analysis Model

With the advancements of computational tools and the demand of shortening product design to market, engineers increasingly rely on mathematical and simulation models. These models can provide a flexible and cheap means to explore and examine design alternatives before physical deployment. With this fundamental paradigm shift, product development is moving toward an engineering process where decisions are heavily based on computational simulations with decreasing physical experiments.

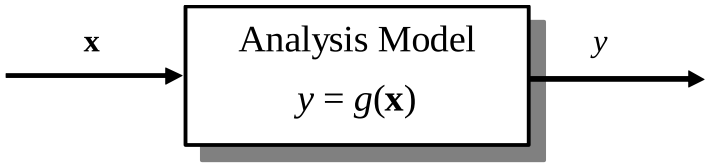

Figure 1.5 A Design/Analysis Model

An analysis model is shown in Fig. 1.5 and is given by

$$y = g(\mathbf{x}) \tag{1.1}$$

In Eq. 1.1, **x** is a vector of input variables. **x** may contain design variables (e.g. the diameter of a shaft) that can be controlled and changed during the design process, or design parameters (e.g. the temperature of the environment) that can not be controlled. In general, **x** is the mixture of design variables and design parameters. y is an output or response variable which is dependent on **x**. y is usually a design performance, such as the cost and maximum stress.

$g(\cdot)$ is the functional relationship between input **x** and output y. In complex engineering design, $g(\cdot)$ may not have an analytic formula. The output is obtained through numerical calculations or simulations. This kind of model is usually called a *black-box* model. Examples of black-box models include those of finite element analysis, dynamics simulation, and computational fluid dynamics. In product development such as a vehicle design, sophisticated engineering computer models are eminent. Different from a scientific model that is to fit extant data, an engineering model is primarily used to predict future performances (behaviors) before a physical product is made.

Analysis models are important for many reasons. (1) Significant upfront design decision-making occurs prior to the availability of physical prototypes. Such design-making relies heavily on the predictions of design performances from the models. (2) Physical testing can be expensive, time consuming, harmful, or even, in some situations, prohibitive. (3) Engineers use models to gain some insights into certain phenomena, which may be lacking from physical experiment due to measurement system limitations or its practicality.

In this class, we will focus on *model-based engineering design* where engineers use analysis models to predict product performances and make design decisions.

1.5 Where Does Uncertainty Come From?

Uncertainty can be viewed as the difference between the present state of knowledge and the complete knowledge (see Fig. 1.6). In the context of model-based design, uncertainty is the difference between the model prediction and reality. Uncertainty is usually classified into *aleatory* and *epistemic* types.

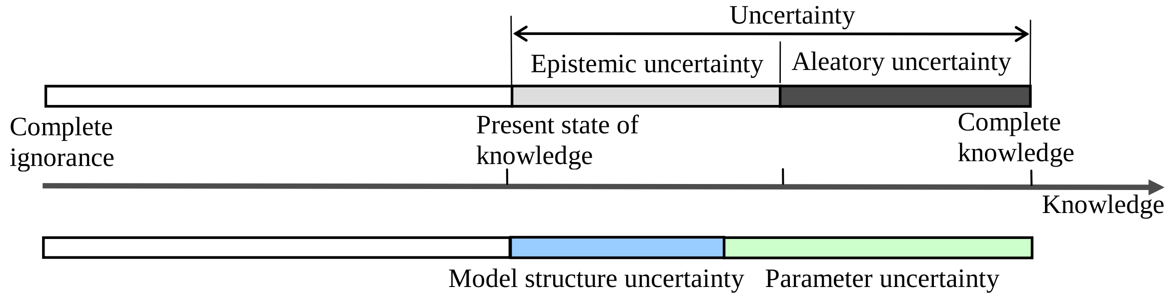

Figure 1.6 Uncertainty types

Aleatory uncertainty, also termed as objective or stochastic uncertainty, describes the inherent variation associated with a physical system or environment. This kind of uncertainty arises from complex physical phenomena, including variations in temperature, material properties, usage conditions, dimensions of a product caused by manufacturing imprecision, etc. Since uncertainty is a result from *natural variability*, aleatory uncertain is *irreducible*. Aleatory uncertainty is usually modeled as randomly distributed quantities that can take values in an established or known range, but the exact values will vary by chance from unit to unit or from time to time.

Epistemic uncertainty is derived from some level of ignorance or incomplete information about a physical system or environment. Epistemic uncertainty is subjective in nature and arises primarily from limited knowledge. The key feature is that the fundamental source of epistemic uncertainty is incomplete information or incomplete knowledge of some characteristics of the system or the environment. In short, epistemic uncertainty is due to the lack of knowledge. Epistemic uncertainty is reducible. The degree of uncertainty can be reduced if more knowledge is acquired or more data are collected.

Uncertainty is also conveniently classified into *parameter uncertainty* and *model structure uncertainty*. *Parameter uncertainty* is due to limited information or the inherent variation in the physical system or environment in estimating the characteristics of parameters. As shown in Fig. 1.6, uncertainty associated with a parameter can be aleatory (due to the inherent variation) or epistemic (due to limited information). For example, if the diameter of a shaft varies around its nominal value within the specified tolerance with a normal distribution, the parameter uncertainty associated with the diameter is aleatory. In another example, engineers do not have enough information about the coefficient of friction between two materials. What they can estimate is that the coefficient of friction is within a range between 0.1 and 0.35. But they do not know how the values of the coefficient of friction are distributed within the range. Due to the lack of knowledge, the parameter uncertainty associated with the coefficient of friction is epistemic. If engineers perform more analyses or experiments, the estimate of coefficient of friction will be more precise and the range will be narrower or reduced to a single value. The epistemic parameter uncertainty will then be reduced or eliminated.

Model structure uncertainty is the uncertainty in the model structure itself, including uncertainty in the validity of the assumptions underlying the model. The uncertainty associated with a model structure is a special type of epistemic uncertainty, which comes from assumptions or a lack of knowledge in the model building process.

The classification of uncertainty is summarized in Fig. 1.7.

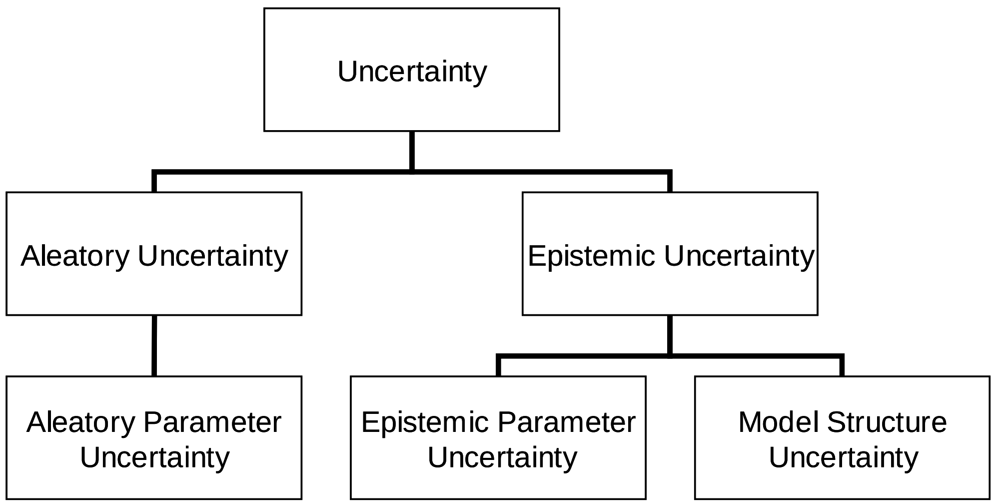

Figure 1.7 Classification of Uncertainty

To better understand the concept of uncertainty, let us look at a simple beam design example (see Fig. 1.8). The design variables that are to be determined are the cross-sectional dimensions, including widths b_1 and b_2, heights h_1 and h_2, and lengths l_1 and l_2. A vertical external force P applies at the tip of the beam. The yield strength of the beam material is S.

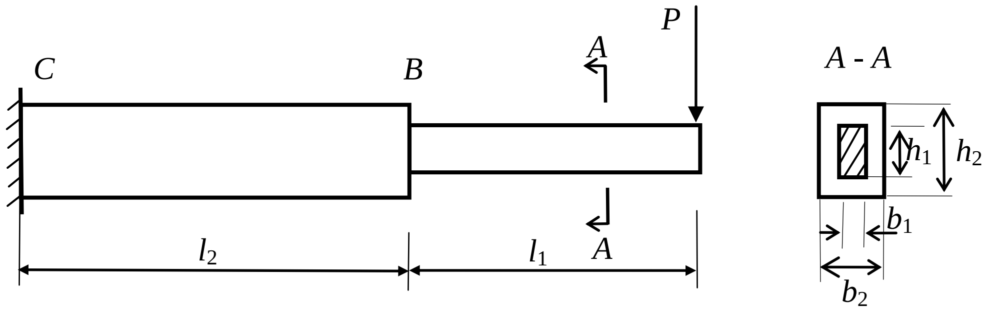

Figure 1.8 A Cantilever Beam

To make the design feasible, it is necessary to calculate the maximum stress $\sigma_{\max}$ and make sure that it is less than the yield strength *S*. The analytical model $y = g(\mathbf{x})$ for the design margin is defined by the difference between the strength and stress, namely,

$$y = g(\mathbf{x}) = S - \sigma_{\max} = S - \frac{6P(l_1 + l_2)}{b_2 h_2^2} \tag{1.2}$$

where $\mathbf{x} = \left[b_1, b_2, h_1, h_2, l_1, l_2, S\right]$ and the maximum stress $\sigma_{\max}$ is derived from the basic beam theory as

$$\sigma_{\max} = \frac{6P(l_1 + l_2)}{b_2 h_2^2} \tag{1.3}$$

Parameter uncertainty: Due to the manufacturing imprecision, the dimension variables b_2, h_2, l_1, and l_2 in Eq. 1.2 are random variables, varying within the tolerance around their nominal values. All the dimension variables are normally distributed, and the parameter uncertainty associated with them is aleatory. Due to the varying operational environment, the external force *P* is also an uncertain variable. If there are adequate data (samples), *P* can be described mathematically with a random distribution. In this case, *P* has aleatory parameter uncertainty. However, if the data are scarce, *P* may not be precisely modeled by a random distribution. Then epistemic uncertainty also exits. Similarly, the material property, the strength *S*, is also subject to uncertainty. For the same reason, the parameter uncertainty of *S* may be either aleatory or epistemic. Because of the input parameter uncertainties in **x**, the model prediction *y* is also subject to uncertainty.

Model structure uncertainty: Eq. 1.2 is derived from the basic beam theory based on several idealized assumptions, such as (1) the material is isotropic and homogenous; the material is also linearly elastic; (2) plane sections remain plane under a load; (3) the moduli of elasticity in tension and compression are identical; and (4) the support of the beam at *C* is perfectly rigid. The assumptions may not be completely valid, and therefore the prediction of the design margin in Eq. 1.2 will be different from the true value. This indicates the existence of model structure uncertainty. Model structure uncertainty is a special type of epistemic uncertainty. If a more sophisticated model is used, the prediction will be closer to the reality, and the model uncertainty will then be reduced.

It is worthwhile to note that numerical errors also exist in model-based predictions. If numerical methods such as finite element analysis are used to solve the maximum stress $\sigma_{\max}$, the solution may not be identical to the solution from the theoretic model in Eq. 1.3. The numerical error is the difference between the numerical solution and the analytical solution (accurate solution).

The above concepts are further demonstrated by the following vehicle crashworthiness design example (Fig. 1.9). Finite element models play an integral part in the vehicle crashworthiness design, which involves uncertain parameters such as those of geometry (shape, thickness, and tolerances), material properties (elasticity, yield strength, and damping), and loading. Some parameters (e.g. tolerances) are random variables with aleatory uncertainty and their probability distributions are precisely known. Other parameters are epistemically uncertain because the knowledge about them is imprecise. For example, the most significant epistemic parameter uncertainty is that of the contact resistance, which is assumed to lie in an interval due to a lack of knowledge [5]. Another significant uncertainty is the model structure uncertainty, a special type of epistemic uncertainty, caused by the linear approximation for stress and strain and by other assumptions [5, 6]. With the model structure uncertainty, proving ground tests (Fig. 1.9) are required to verify that the model-based design meets the mandated crashworthiness standards. There are also many sources of uncertainty in the vehicle manufacturing process and proving ground tests that, in turn, induce experimental uncertainty in the test results, which also contain both aleatory and epistemic uncertainties.

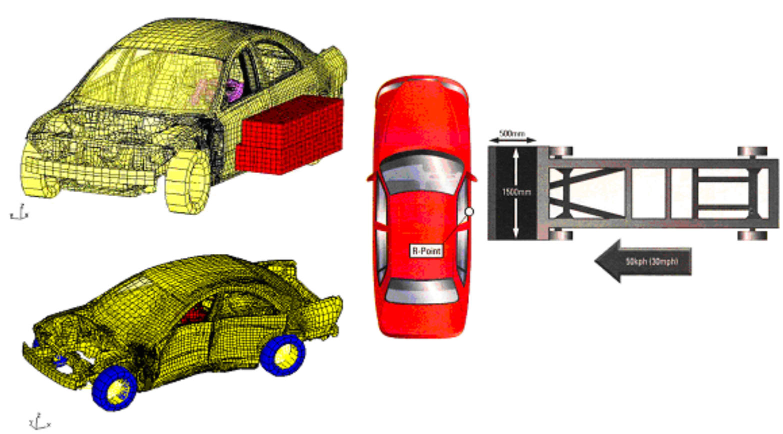

Figure 1.9 Vehicle Crash Simulation (left) and Proving Ground Test (right) (Courtesy of Ford Motor Company)

Probability theory is commonly used to quantify aleatory uncertainty. Quantifying and managing epistemic uncertainty (epistemic parameter uncertainty and model structure uncertainty) needs more advanced mathematical theories [7]. It is still an ongoing research topic in both academia and industry. In this class, we will primarily focus on aleatory parameter uncertainty.

1.6 What is Probabilistic Engineering Design?

As discussed above, engineers increasingly rely on analysis (simulation) models. Unlike scientific models of nature developed to fit extant data, engineering analysis models are intended to predict future performance of systems. Uncertainties are considerable, and they cannot be controlled or minimized beyond modest limits. Thus, it is important to quantify and manage uncertainty inherent in engineering design. Probabilistic engineering design is a design methodology that meets such a need.

Probabilistic engineering design is a mathematically based design methodology for producing high quality products. The features of probabilistic engineering design include

- Using probability theory to quantify and treat uncertainties
- Accommodating uncertainties and mitigating the effects of uncertainties
- Ensuring high robustness, reliability, and safety
- Integrating optimization with mathematical or CAE (Computer Aided Engineering) simulation models
- Combining Design of Experiments in many engineering applications

Typical methods of probabilistic engineering design include

- Reliability-based design (RBD)
  Reliability-based design seeks a design that has a probability of failure less than some acceptable (invariably small) value and therefore ensures that the events that lead to catastrophe are extremely unlikely. The focus is the higher reliability (safety) and lower risk.

- Robust design
  Robust design is a method for improving the quality of a product through minimizing the effect of uncertainty without eliminating the causes of uncertainty. The focus is the robustness of the product performance.

- Design for Six Sigma (DFSS)
  Design for Six Sigma is a comprehensive approach to product development that links business and consumer needs to critical product attributes, product functions, detailed designs, tests, and verification. The focus is the product quality, customer satisfaction, and competitiveness. RBD and robust design are usually employed in DFSS.

1.7 The Effects of Uncertainty on Product Performance

Uncertainty associated with parameters, model structures, and numerical errors has a significant impact on design performances. The ignorance or inappropriate treatment of uncertainty may lead to

- Erroneous decision-making,
- Low quality, robustness, reliability, safety,
- High risk,
- High cost of product-life cycle,
- Costly warranty,
- Over-designed (conservative) products,
- Low customer satisfaction, and
- Catastrophic consequences

For example, if a product is not robust, the product performance will be sensitive to the variation of system inputs. As a result, small variations of system inputs such as the imprecision of manufacturing may lead to a large variation in product performance. A large variation in performance means low quality and will consequently results in low customer satisfaction. In addition, if a product is not reliable, the chance of failure will be relatively high. Catastrophic events may occur when the product fails.

1.8 Why Does An Engineer Need to Know Probabilistic Engineering Design?

Traditionally, engineering design has been performed on a deterministic basis as if everything could be calculated with certainty through formulae or simulations that modeled nature with absolute precision. Due to this belief, engineers are typically trained through the courses where the analysis models are formulated ideally in such a way that there are no variations. Given a deterministic input, there always exists a deterministic output.

As we have seen previously, uncertainty exists in every engineering system. It impacts the product performance significantly. A small variation of system input may cause a huge quality loss. The ignorance of uncertainty may lead to catastrophic failure events. To accommodate uncertainty, the common practice is the use of a factor of safety. It has long been customary for engineers to modify their design with “arbitrary” factors of safety so that bridges would not collapse and machines would not break down. It is evident that the use of factors of safety may be either risky (under-designed) or conservative (over-designed).

With the intensive requirement of high product quality, the more complex computer simulations are increasingly being used, and more complicated decision making is required during a design process. The need of uncertainty consideration in engineering design has become imperative. Probabilistic and statistical based design methods, such as Design of Experiments (DOE), robust design, reliability-based design, and Design for Six Sigma (DFSS), have been used in industry to meet such a need. A typical example is the vehicle development process, where vehicle program managers are continually challenged with tasks with the presence of uncertainties. The typical tasks include integrating uncertain information across a large number of functional areas, assessing program risk relative to business goals, and then making program-level decisions. In the mean time, engineers struggle to develop design alternatives facing with uncertainties in design and analysis models, manufacturing processes, and environment. They must provide the program managers with credible, timely, and robust estimates of design related vehicle performance [8]. For many engineers in other industries, applying nondeterministic approaches to handle uncertainty has also become a part of their routine job.

1.9 Uncertainty Management

In this class, we will discuss how to deal with uncertainty in engineering design at three complementary levels – modeling, analysis, and design. The three levels are illustrated in Fig. 1.10.

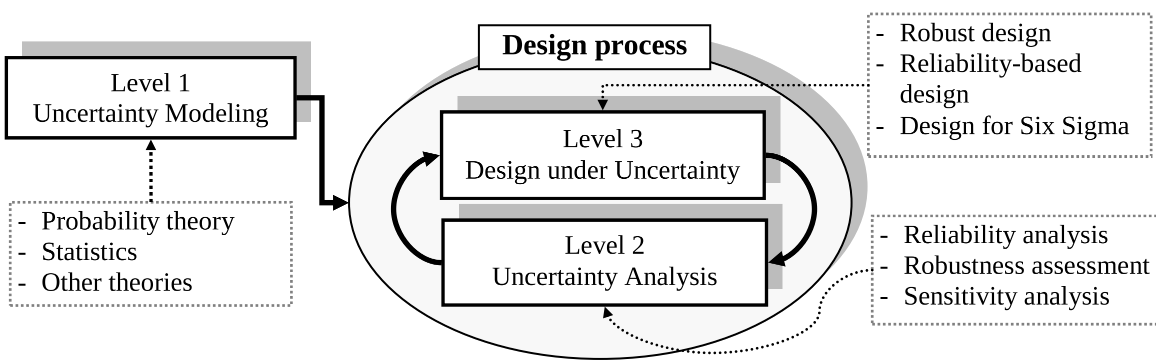

Figure 1.10 Managing Uncertainty in a Design Process

1. Level 1 – Uncertainty modeling

The task of uncertainty modeling is to quantify uncertainty mathematically. Probability theory is commonly used for this task. An uncertain quantity is described by a random variable and is characterized by a probability distribution. Since the distribution is usually obtained from statistical data, statistics is used to formulate the distribution. The mathematical structures of uncertain variables at uncertainty modeling level then provide the input to uncertainty analysis at the next level.

2. Level 2 – Uncertainty analysis
The task of uncertainty analysis is to quantify the uncertainty of design performance (model output) given the uncertainty of model input. The uncertainty of model input is modeled at the above modeling level. Uncertainty analysis helps engineers understand how uncertainty impacts design performance and provide them with tools to evaluate important design characteristics, such as reliability and robustness. The knowledge from uncertainty analysis will then be used at the next design level for managing and mitigating the effects of uncertainty.

3. Level 3 – Design under uncertainty
The task of design under uncertainty is to mitigate the effects of uncertainty by making appropriate decisions. Depending on design needs, the focus may be on the reliability (safety), robustness, or quality. To make the design cost-effective, the common practice is to determine optimal design variables at the design stage without eliminating the causes of uncertainty. The reason is that in many cases, eliminating uncertainty causes is very expensive. It requires high precision manufacturing and strict quality control. As indicated in Fig. 1.10, design under uncertainty is an iterative process. During this process, the design is continually updated until satisfactory design is achieved. Uncertainty analysis is performed for each updated design. Therefore, the design process repeatedly calls uncertainty analysis.

1.10 Concluding Remarks

We have reviewed the general engineering design process and introduced important concepts of uncertainty. In this class, we will primarily use probability theory to model aleatory uncertainty in probabilistic engineering design. The methods of probabilistic engineering design introduced in this class can be used in stages of conceptual design, preliminary deign, and detail design. In the following chapters, we will first present the basic probability theory from an engineering perspective, for readers who may or may not be familiar with probability theory. Then we will discuss the basic probabilistic analysis and design methods that are commonly practiced in industry. Equipped with the basic tools provided in this class, students will have a basic working knowledge for uncertainty analysis and design under uncertainty.

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