Homework 2

1. During an experiment in class, a number of paper clips were bent until each failed. The numbers of cycles to failure were recoded as shown below.

19.5	18	12	13	11	40	17	18	12	14	18.5	17.5
20	15	16	18.5	29	21	23	21	19	24	18	33
28	29	58	37								

    (1) Calculate the average of the cycles to failure.

    (2) Calculate the standard deviation of the cycles to failure.

    (3) Calculate the median of the cycles to failure.

2. The angular velocity X of a shaft varies randomly with the following probability density

    $$f(x) = \begin{cases} k & 5.5 \text{ rad/s} \leq X \leq 6 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

    (1) What is k ?
    (2) What the mean value of the angular velocity?
    (3) What is the standard deviation of the angular velocity?
    (4) What the median of the angular velocity?
    (5) What is the probability $\Pr\{X > 5.8 \text{ rad/s}\}$?
    (6) What is the 95% percentile value of the angular velocity?

3. A system consists of two electronic components as shown in Fig. 1.
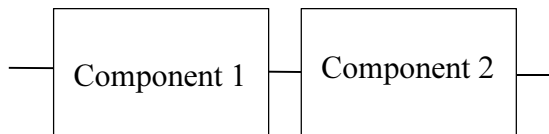

    Fig 1. A system with two components

    The lives of the two components T_1 and T_2 follow the following distributions:

    $$f_{T_1}(t) = \lambda_1 e^{-\lambda_1 t} \quad (t \geq 0)$$
    $$f_{T_2}(t) = \lambda_2 e^{-\lambda_2 t} \quad (t \geq 0)$$

    where $\lambda_1 = 0.00001$/hr and $\lambda_2 = 0.00002$/hr . T_1 and T_2 are independent.

    (1) Determine the joint PDF of T_1 and T_2.

    (2) Determine the reliability of the system after the system is in operation for 1000 hr.

4. A joint of a mechanism can be considered as a journal bearing. As shown in Fig. 2, the radius of the bearing r_B is greater than that of the journal r_J. Their difference is called a clearance, and $r = r_B - r_J$. As indicated in Fig. 3, the position (X, Y) of the center of the journal relative to the center of the bearing is within a circle of radius r. This circle is called a clearance circle.
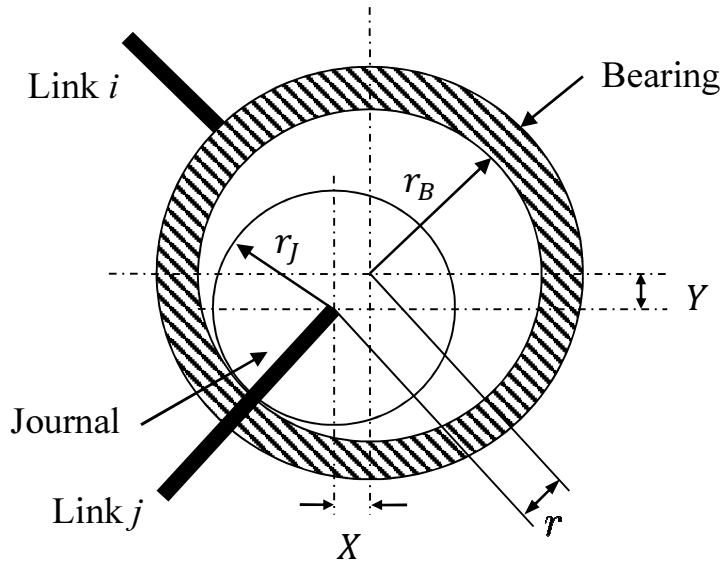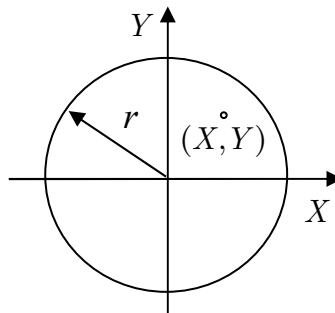

Fig. 2 Joint clearance

Fig. 3 Clearance circle

Assume that the clearance r is known and that the position (X, Y) is uniformly distributed within the clearance circle $\sqrt{X^2 + Y^2} \leq r$. (The PDF is constant within the clearance circle $\sqrt{X^2 + Y^2} \leq r$.) Determine
(1) The joint probability density function (PDF) of X and Y
(2) The PDFs of X and Y
(3) The means of X and Y
(4) The variances of X and Y
(5) The coefficient of correlation between of X and Y