Homework 3

1. The yield strength, Y, of the material used for a mechanical component, is normally distributed with $Y \sim N(200, 35^2)$ MPa. The stress of the component is also normally distributed. The field statistical data of the stress X are collected and are shown in the following table.

(1) What is the deterministic factor of safety?
(2) What is the probability of failure?
(3) Among 10000 components, how many components are expected to fail?

Table 1 Observations of Stress X (MPa)

77.19	19.94
59.11	99.65
61.01	62.03
86.09	83.18
51.25	80.85
94.52	75.53
89.22	49.90
54.45	51.32
68.35	65.63
45.30	45.35

2. The strength of a mechanical component follows a lognormal distribution with a mean of 200 MPa and standard deviation of 25 MPa.

1) What is the probability that the component will have strength less than 180 MPa?
2) What is the 95% percentile strength?

3. A number of CAE simulations are performed for the engine slap noise for a vehicle engine design. Through the Design of Experiments (DOE) and simulations, the model of the noise Y (dB) is created with respect to design variables X_1, X_2, and X_3 as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

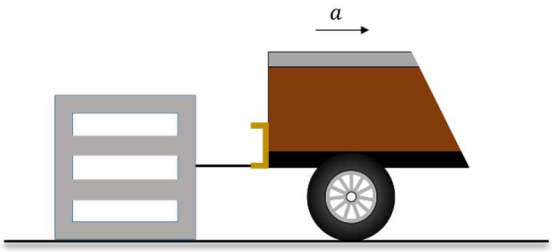
in which $\beta_0 = 53.8974$, $\beta_1 = 0.0817$, $\beta_2 = -0.0283$, and $\beta_3 = -3.2312$; X_i $(i = 1, 2, 3)$ are random variables, which are given in the following table.

Table 2 Random Variables

Random Variables	Mean	Standard Deviation	Distribution
X_1 - Clearance	65	5	Normal
X_2 - Length	22.5	3	Normal
X_3 - Offset	0.9	0.1	Normal

(1) If the design requirement for the noise is $Y < 57$ dB. What is the probability that the design satisfies the requirement?
(2) If the reliability (the probability of requirement satisfaction) is not satisfactory, how do you suggest improving the design?

4. A concrete block of 10,000 kg is towed by a vehicle through a cable. The vehicle starts from rest with an acceleration of $a \sim N(3, 0.1^2)$ m/s^2, and the static and kinetic coefficients of friction between the block and ground are $\mu_s \sim N(0.35, 0.03^2)$ and $\mu_k \sim N(0.2, 0.02^2)$, respectively. The strength of the cable is $S \sim N(100, 15^2)$ kN. Assume all the random variables are independent. Determine the probability that the cable would break.



5. The weight of the crate follows a normal distribution $W \sim N(1500, 180^2)$ kN. The allowable tensions of the two cables are also normally distributed with $S_1 \sim N(1200, 80^2)$ kN and $S_2 \sim N(2200, 100^2)$ kN, respectively. The three random variables are independent. Determine the reliability of the system. (Consider only the two cables. Neglect the weight of the pulley.)