Homework 7

Sensitivity Analysis and DOE

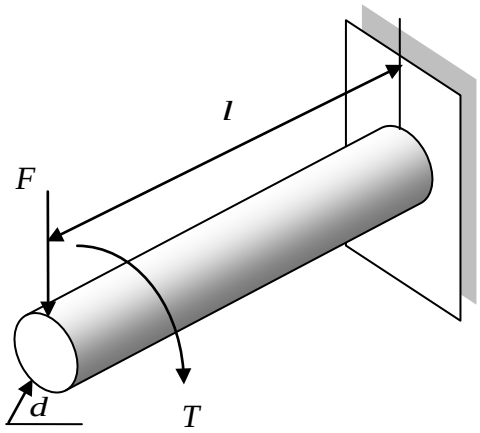
1. **Sensitivity Analysis**

The limit-state function of a shaft in a speed reducer is defined by the difference between the strength and the maximum equivalent stress. It is given by

$$g(\mathbf{X}) = S - \frac{16}{\pi d^3}\sqrt{4F^2l^2 + 3T^2}$$

where

$d = 39$ mm, the diameter of the shaft
$l = 400$ mm, the length of the shaft
$F =$ the external force
$T =$ the external torque
$S =$ the yield strength



The distributions of the independent random variables are given below.

Table 1 Distributions

Variables	Mean	Std	Distribution
External force F	2000 N	220 N	Normal
Torque T	450 N·m	50 N·m	Normal
Strength S	250 MPa	30 MPa	Normal

Use the MPP result from Homework 6 to calculate the reliability sensitivity with respect to the means and standard deviations of the three random variables.

2. DOE

Piston slap is unwanted vehicle engine noise caused by piston secondary motion. For understanding the problem, DOE is performed. Three parameters (clearance, length, and offset) are considered as design variables. The DOE matrix and the experimental results from a 2-level full factorial design for the noise are obtained as shown in Table 1.

Table 1 DOE Matrix and Experimental Results

Experiments	Clearance: X_1	Length: X_2	Offset: X_3	Noise: Y (dB)
1	15	21	0.5	55.17
2	85	21	0.5	62.37
3	15	24	0.5	52.05
4	85	24	0.5	65.33
5	15	21	1.3	57.11
6	85	21	1.3	55.27
7	15	24	1.3	53.98
8	85	24	1.3	58.22

1) Create a DOE model with the following form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3$$

2) A design is given in Table 2. A failure is defined as the event when the noise exceeds 63 dB. Use Monte Carlo Simulation to calculate the probability of failure and reliability of the design.

Table 2 Distribution of Random Variables

Random variables	Distribution
X_1	Normal, mean = 80, std =0.5
X_2	Normal, mean = 23, std =1
X_3	Uniform, a = 0.9, b = 1

Solution: $p_f = 1.957 \times 10^{-4}$ (Your solution may be slightly different.)