

## Homework 8 Optimization

### 1. MPP search by optimization

Find the MPP using optimization for the problem in Homework 5. The original problem is stated as follows:

Consider the limit-state function of a shaft in a speed reducer defined by

$$Y = g(\mathbf{X}) = S - \frac{16}{\pi d^3} \sqrt{4F^2 l^2 + 3T^2} \quad (1)$$

where  $d = 39$  mm is the diameter of the shaft,  $l = 400$  mm is the length of the shaft,  $F$  is the external force,  $T$  is the external torque, and  $S$  is the yield strength.

### Distributions of Random Variables

Variables	Mean	Std	Distribution
External force $F$	2000 N	220 N	Normal
Torque $T$	450 N·m	50 N·m	Normal
Strength $S$	250 MPa	30 MPa	Normal

### 2. Optimization

(1) Solve the following optimization problem graphically.

$$\min f(\mathbf{x}) = x_1^2 + x_2^2 - 2x_1 - 2x_2$$

$$\text{s.t. } g_1(\mathbf{x}) = x_1 + x_2 - 4 \leq 0$$

$$g_2(\mathbf{x}) = 2 - x_1 \leq 0$$

$$x_1, x_2 \geq 0$$

(2) Solve the above problem by a numerical method. You may use any optimization software. Attach the code you used.

### 3. Optimization for worse case analysis

If the three variables  $F$ ,  $T$ , and  $S$  in problem 1 are interval values. Find the range of  $Y$  defined in Eq. (1). Use the following lower and upper bounds for the interval variables:

$$X_{\min} = \mu_X - 2\sigma_X, \quad X_{\max} = \mu_X + 2\sigma_X$$

where  $X$  is  $F$ ,  $T$ , or  $S$ .