# Estimate π by Monte Carlo Simulation

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We show how to use Monte Carlo simulation (MCS) to estimate π.

## 1. Approach

As shown in Figure 1, the radius of the circle is $r$. Let $X$ and $Y$ be independently and uniformly distributed on $[-r, r]$. Then their probability density functions (PDFs) are

$$f_X(x) = \frac{1}{2r} \text{ where } -r \le x \le r \tag{1}$$

and

$$f_Y(y) = \frac{1}{2r} \text{ where } -r \le y \le r \tag{2}$$

respectively.

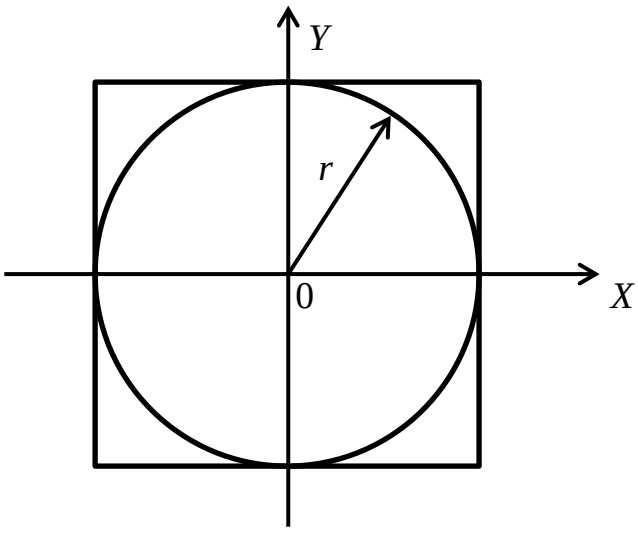


**Figure 1 A Square and A Circle**

Thus, their joint PDF is

$$f_{XY}(x, y) = f_X(x) f_Y(y) = \frac{1}{4r^2} \text{ where } -r \le x, y \le r \tag{3}$$

The probability that point $(X, Y)$ falls into the circle is given by

$$P = \iint_{x^2+y^2 \le r^2} f_{XY}(x, y) dx dy = \frac{1}{4r^2} \iint_{x^2+y^2 \le r^2} dx dy = \frac{\pi r^2}{4r^2} = \frac{\pi}{4} \tag{4}$$

Now we draw $n$ random points $(X, Y)$ uniformly in the square, and suppose $m$ points fall into the circle. Then the probability $P$ can be estimated by

$$P \approx \frac{m}{n} \tag{5}$$

From Equations (4) and (5), we have

$$\pi \approx \frac{4m}{n} \tag{6}$$

With a higher sample size $n$, we will get higher accuracy for estimating π. When $n \to \infty$ , the estimated π will approach its true value.

## 2. Simulation results

As shown in Figure 2, when $n = 100$, 79 $(m = 79)$ points fall into the circle. $\pi \approx \frac{4(79)}{100} = 3.16$. If we increase $n$ to $10^6$, a more accurate result is obtained, and $\pi \approx 3.144$.

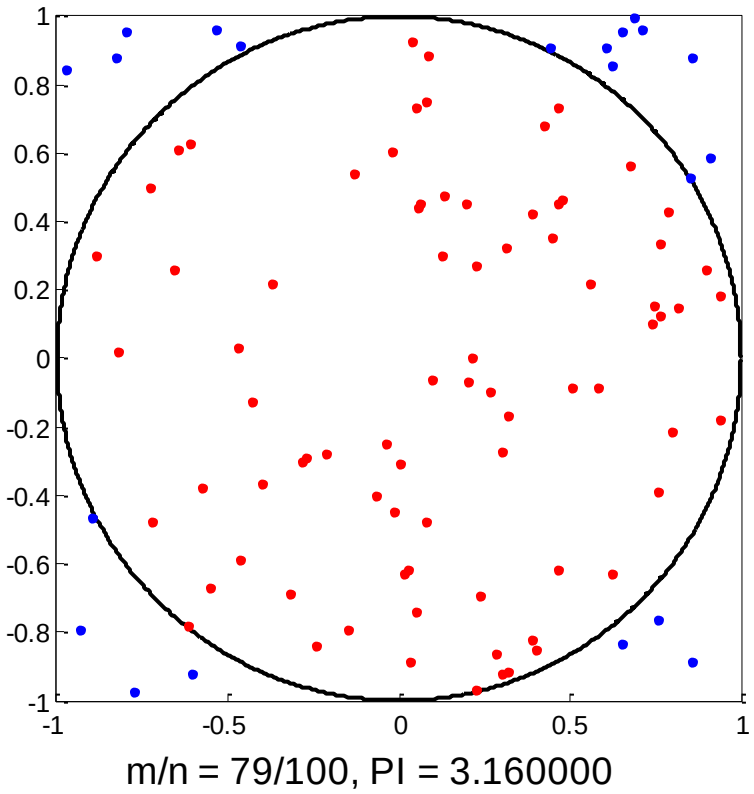


**Figure 2 Simulation Result When $n = 100$**

### 3. Matlab code

```
%Estimate PI Monte Carlo Simulation
%By Xiaoping Du, 02/19/2014
%Center of the circle = (0,0)
%Radius of the circle = 1
clc;close all; clear all;
n=input('Enter number of samples:');   %# of samples
rand('seed',108);                      %Initialize the random generator
x=2*rand(n,1)-1;                       %Samples of x
y=2*rand(n,1)-1;                       %Samples of y
m=sum(x.^2+y.^2<1);                    %# of samples in the circle
pi_est=4*m/n;                          %Estimate of PI


%Display
rectangle('Position',[-1,-1,2,2]);     %Draw the square
hold on;
theta=linspace(0,2*pi,1e3);            %Draw the circle
rho=ones(1,1e3);
[xc,yc] = pol2cart(theta,rho);
plot(xc,yc,'k-','linewidth',2);
axis square;

inside=find(x.^2+y.^2<1);              %Get samples in the circle
outside=find(x.^2+y.^2>=1);            %Get samples outside the circle
plot(x(inside),y(inside),'r.');        %Plot samples in the circle
plot(x(outside),y(outside),'b.');      %Plot samples outside the circle

text(-0.8,-1.2,['m/n = ',num2str(m),'/',num2str(n),...
    ', PI = ',num2str(pi_est,'%8.6f')],'FontSize',14); %Display the result
```