A joint of a mechanism can be considered as a journal bearing. As shown in Fig. 1, the radius of the bearing r_B is greater than that of the journal r_J . Their difference is called a clearance, and $r = r_B - r_J$. As indicated in Fig. 2, the position (X, Y) of the center of the journal relative to the center of the bearing is within a circle of radius r. This circle is called a clearance circle.

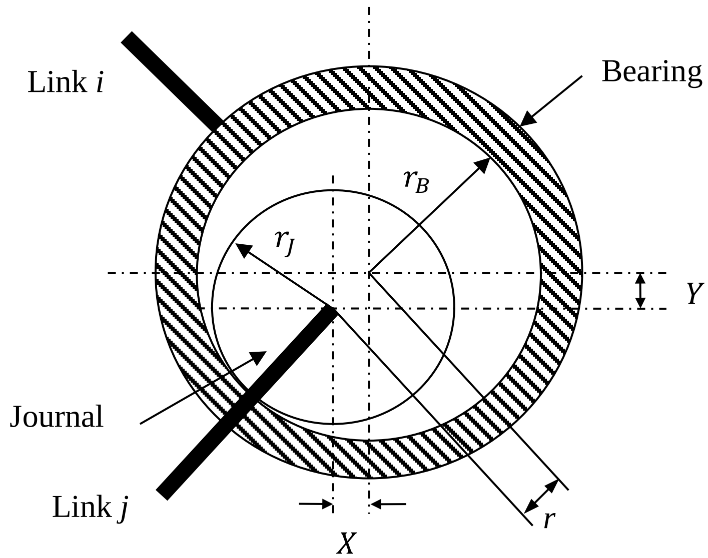

Fig. 1 Joint clearance

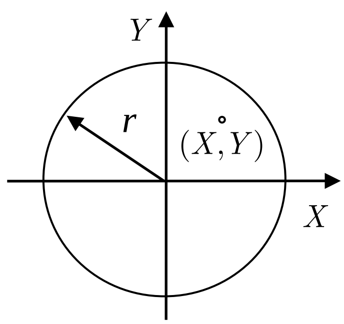

Fig. 2 Clearance circle

Assume that the clearance is known and that the position (X, Y) is uniformly distributed within the clearance circle $\sqrt{X^2 + Y^2} \leq r$. Determine

(1) The joint PDF and CDF of *X* and *Y*

(2) The PDFs of *X* and *Y*

(3) The means of *X* and *Y*

(4) The variances of *X* and *Y*

(5) The coefficient of correlation between of *X* and *Y*

Solution

(1)

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{1}{\pi r^2} & x^2 + y^2 \le r^2 \\ 0 & \text{otherwise} \end{cases}$$

(2)

$$f_X(x) = \begin{cases} \dfrac{2\sqrt{r^2 - x^2}}{\pi r^2} & , -r \le x \le r \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \dfrac{2\sqrt{r^2 - y^2}}{\pi r^2} & , -r \le y \le r \\ 0 & \text{otherwise} \end{cases}$$

(3)

$$\mu_X = 0 \,, \mu_Y = 0$$

(4)

$$\sigma_X^2 = \frac{r^2}{4}, \sigma_Y^2 = \frac{r^2}{4}$$

$$\sigma_Y^2 = \frac{r^2}{4}$$

(5) 0