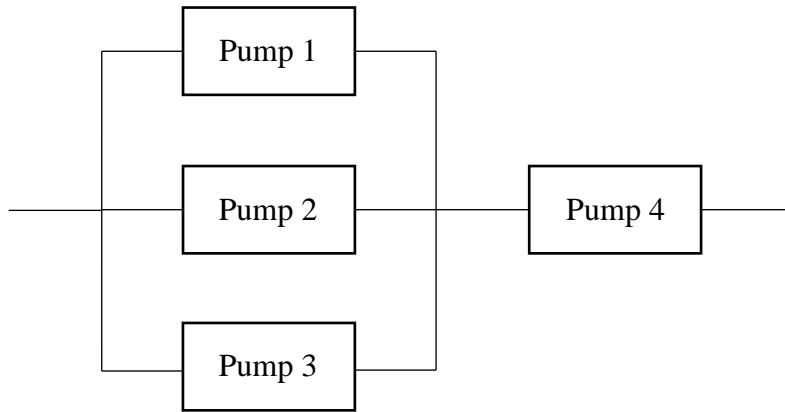


A pumping system on an oil field consists of four pumps as shown below. The reliabilities of the individual pumps are  $R_1 = 0.96, R_2 = 0.94, R_3 = 0.88,$  and  $R_4 = 0.94,$  and the states of the pumps are independent. (1) Calculate the reliability and the probability of failure of the system. (2) Assume that the cost of improving reliability of each pump is the same. If a higher system reliability is desired and the reliability of only one pump can be improved due to the cost concern, which pump should be improved in terms of reliability?



**Solution**

The subsystem, which is composed of components 1, 2 and 3, is a parallel system. Its reliability is given by

$$R_{123} = 1 - (1 - R_1)(1 - R_2)(1 - R_3) = 0.997$$

The entire system is a series system composed of subsystem 123 and component 4, and therefore its reliability is

$$R = R_{123}R_4 = [1 - (1 - R_1)(1 - R_2)(1 - R_3)]R_4 = 0.9397$$

$$\frac{\partial R}{\partial R_1} = (1 - R_2)(1 - R_3)R_4 = 0.0068$$

$$\frac{\partial R}{\partial R_2} = (1 - R_1)(1 - R_3)R_4 = 0.0045$$

$$\frac{\partial R}{\partial R_3} = (1 - R_1)(1 - R_2)R_4 = 0.0023$$

$$\frac{\partial R}{\partial R_4} = [1 - (1 - R_1)(1 - R_2)(1 - R_3)] = 0.9997$$

Among all the derivatives,  $\frac{\partial R}{\partial R_4}$  is the largest. Therefore the reliability of pump 4 needs to be improved.