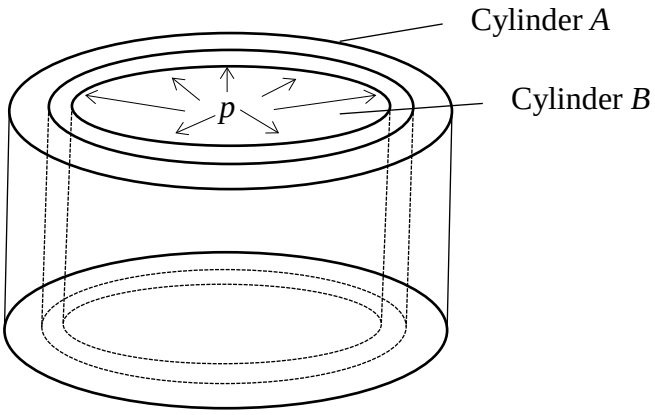
A pressure vessel that consists of two hollow cylinders is shown in the figure. The pressure vessel is subjected to an interval pressure p. If only one cylinder is subjected to p along, the reliabilities of cylinders A and B are 0.996 and 0.995, respectively. If both cylinders operate as shown in the figure, the probability that either cylinder fails is 0.002, and the probability that both of the cylinders work properly is 0.98. If one cylinder fails, the other cylinder will continue to operate and resist the entire pressure, what is the reliability of the pressure vessel?



Solution

Define events as follows.

A_S: Cylinder A fails when it works alone

B_S: Cylinder B fails when it works alone

A_B: Cylinder A fails when both cylinders resist p

B_A: Cylinder B fails when both cylinders resist p

C: Both cylinders resist p

$$P(A_S) = 1 - 0.996 = 0.004$$
$$P(B_S) = 1 - 0.995 = 0.005$$
$$P(A_B) = 0.002$$
$$P(B_A) = 0.002$$
$$P(C) = 0.98$$
$$R = \Pr\{\text{Both cylinders work} \cup A \text{ works and } B \text{ fails} \cup B \text{ works and } A \text{ fails}\}$$
$$R = \Pr\{C \cup \overline{A_S} \cap B_A \cup \overline{B_s} \cap A_B\} = \Pr\{C\} + \Pr\{\overline{A_S} \cap B_A\} + \Pr\{\overline{B_s} \cap A_B\}$$
$$= \Pr\{C\} + \Pr\{\overline{A_S}\}\Pr\{B_A\} + \Pr\{\overline{B_s}\}\Pr\{A_B\}$$
$$= 0.98 + 0.996(0.002) + 0.995(0.002) = 0.9840$$