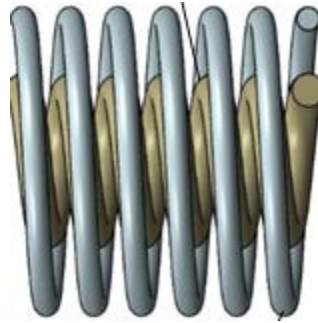A spring constant changes uniformly between 500 N/m and 550 N/m.
(1) Find the PDF and CDF of the spring constant.
(2) What is the probability that the spring constant is greater than 540 N/m?
(3) If the spring is stretched by 1 cm, what is the probability that the spring force is smaller than 5.3 N?

Solution

(1) Let X be the spring constant, $a = 500$, and $b = 550$. The PDF of the spring constant is

$$f(x) = \frac{1}{b-a} = \frac{1}{550-500} = \frac{1}{50}$$

where $500 \le x \le 550$.

The CDF of the spring constant is

$$F(x) = \int_{-\infty}^{+\infty} f(x)\, dx = \int_{a}^{x} \frac{1}{b-a}\, dx = \int_{500}^{x} \frac{1}{50}\, dx = \frac{x-500}{50}$$

where $500 \le x \le 550$.

(2) The probability that the spring constant is greater than 540 N/m is calculated by

$$\Pr(X > 540) = 1 - \Pr(X \le 540) = 1 - F(540) = 1 - \frac{540-500}{50} = 0.2$$

(3) If the spring is stretched by 1 cm, the spring force is equal to $F_S = 0.01X$. The probability that the spring force is smaller than 5.3 N is

$$\Pr(F_S < 5.3) = \Pr(0.01X < 5.3) = \Pr(X < 530) = F(530) = \frac{530-500}{50} = 0.6$$