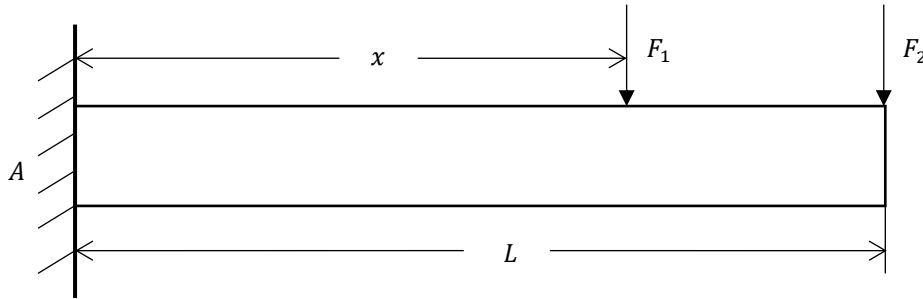


A cantilever beam of span of $L = 1$ m is subjected to a deterministic force $F_1 = 80$ kN and a random force $F_2 \sim N(100, 40^2)$ kN as shown in the figure. The maximum allowable moment at A is $M_{allow} = 260$ kN·m. If the reliability of the beam is at least 0.999, determine the point of action x for force F_1 . HINT: The reliability is the probability that the moment at A is less than M_{allow} .



Solution

The moment at A is

$$M = F_1x + F_2L$$

$$R = \Pr(M < M_{allow}) = \Pr(F_1x + F_2L < M_{allow})$$

$$= \Pr\left(F_2 < \frac{M_{allow} - F_1x}{L}\right) = \Phi\left(\frac{\frac{M_{allow} - F_1x}{L} - \mu_{F_2}}{\sigma_{F_2}}\right)$$

Then

$$x = \frac{M_{allow} - [\Phi^{-1}(R)\sigma_{F_2} + \mu_{F_2}]L}{F_1} = \frac{260 - [\Phi^{-1}(0.999)(40) + 100](1)}{80} = 0.455 \text{ m}$$