The velocity of a car follows the distribution given by

$$f(x) = \begin{cases} k & 50 \text{ mile/hr} \le X \le 60 \text{ mile/hr} \\ 0 & \text{otherwise} \end{cases}$$

- (1) What is k ?
- (2) If the distance between cities *A* and *B* is 165 mile, what is the probability that the travel time from city *A* to city *B* is less than 3 hours?



Solution

(1) Let the velocity be X. Since

$$\int_{-\infty}^{+\infty} f(x) \, dx = \int_{50}^{60} k \, dx = 10k = 1$$
$$k = \frac{1}{10}$$

(2) The velocity should be greater than $\frac{165}{3} = 55$ mile/hr if the travel time between cities *A* to city *B* is less than 3 hours. Then the probability that the travel time from city *A* to city *B* is less than 3 hours is calculated by

$$\Pr(X > 55) = \int_{55}^{+\infty} f(x) \, dx = \int_{55}^{60} \frac{1}{10} \, dx = \frac{1}{10}(60 - 55) = 0.5$$