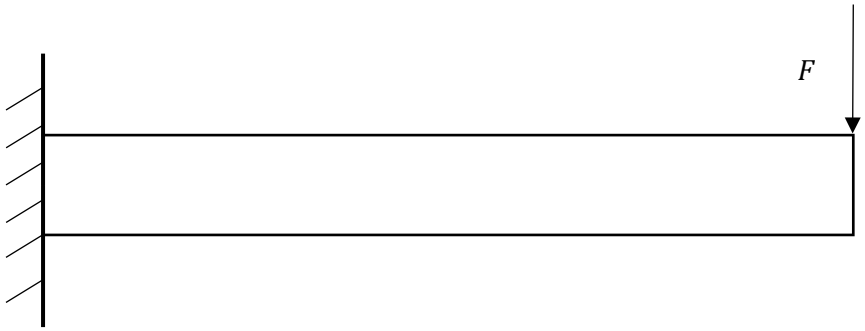
The yield strength of a cantilever beam shown in the figure is lognormally distributed. The mean and standard deviation of the yield strength are 200 MPa and 20 MPa, respectively. If the maximum stress of the beam is 190 MPa, what is the probability that a yield failure will occur?




Solution

Let the yield strength of the cantilever beam be $Y$, which follows a lognormal distribution. We know that $\mu_Y = 200$ and $\sigma_Y = 20$. Then $X = \ln(Y)$ follows a normal distribution; namely, $X \sim N(\mu_X, \sigma_X^2)$.

$$\mu_X = \ln\left(\frac{\mu_Y^2}{\sqrt{\mu_Y^2 + \sigma_Y^2}}\right) = \ln\left(\frac{200^2}{\sqrt{200^2 + 20^2}}\right) = 5.2933$$

$$\sigma_X = \sqrt{\ln\left(\frac{\sigma_Y^2}{\mu_Y^2} + 1\right)} = \sqrt{\ln\left(\frac{20^2}{200^2} + 1\right)} = 0.0998$$

The probability of yield failure is the probability that the yield strength is less than the maximum stress. The probability is then calculated by

$$\Pr(Y < 190) = \Pr(\ln(Y) < \ln 190) = \Pr(X < \ln 190) = \Phi\left(\frac{\ln 190 - \mu_X}{\sigma_X}\right)$$

$$= \Phi\left(\frac{5.2470 - 5.2933}{0.0998}\right) = \Phi(-0.4639) = 0.3214$$