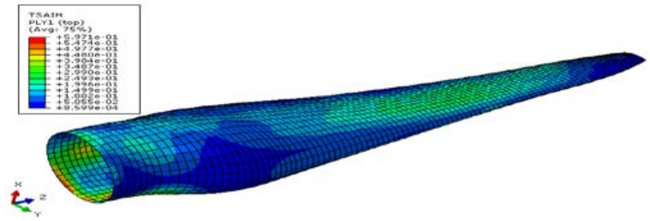
The fatigue life of a turbine blade shown in the figure follows a normal distribution $N(80{,}000, 8{,}000^2)$ cycles.

(1) What is the probability that the blade works without failure for 60,000 cycles?

(2) After the blade has operated for 50,000 cycles successfully, what is the probability that the blade can still continue to work without failure for 10,000 cycles?



Solution

Let the fatigue life be $X \sim N(\mu_X, \sigma_X^2)$, where $\mu_X = 80{,}000$, and $\sigma_X = 8{,}000$.

(1) The probability that the blade works without failure for 60,000 cycles

$$\Pr(X > 60{,}000) = 1 - \Pr(X < 60{,}000) = 1 - \Phi\left(\frac{60{,}000 - 80{,}000}{8{,}000}\right) = 1 - \Phi(-2.5)$$
$$= 0.9938$$

(2) After the blade has operated for 50,000 cycles successfully, the probability that blade can still continue to work without failure for 10,000 cycles

Define the following events:

A: The blade works without failure for 50,000 cycles

B: The blade works without failure for 60,000 cycles

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} = \frac{\Pr(X > 50{,}000, X > 60{,}000)}{\Pr(X > 50{,}000)} = \frac{\Pr(X > 60{,}000)}{\Pr(X > 50{,}000)}$$
$$= \frac{1 - \Pr(X < 60{,}000)}{1 - \Pr(X < 50{,}000)} = \frac{1 - \Phi\left(\frac{60{,}000 - 80{,}000}{8{,}000}\right)}{1 - \Phi\left(\frac{50{,}000 - 80{,}000}{8{,}000}\right)} = \frac{1 - \Phi(-2.5)}{1 - \Phi(-3.75)}$$
$$= 0.9939$$